UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session 2
Fall 2018

Problem 1. **Linearity of Expectation, Independence**
Let X_1, X_2, X_3 be discrete independent random variables with mean 0. Find $\mathbb{E}[(X_1 + X_2)(X_2 + X_3)(X_3 + X_1)]$.

Problem 2. **Poisson Packet Routing**
Packets arriving at a switch are routed to either destination A (with probability p) or destination B (with probability $1 - p$). The destination of each packet is chosen independently of each other. In the time interval $[0, 1]$, the number of arriving packets is $\text{Pois}(\lambda)$.

1. Show that the number of packets routed to A is Poisson distributed. With what parameter?
2. Are the number of packets routed to A and to B independent?

Problem 3. **Poisson Merging**
The Poisson distribution is used to model *rare events*, such as the number of customers who enter a store in the next hour. The theoretical justification for this modeling assumption is that the limit of the binomial distribution, as the number of trials n goes to ∞ and the probability of success per trial p goes to 0, such that $np \to \lambda > 0$, is the Poisson distribution with mean λ.
Now, suppose we have two independent streams of rare events (for instance, the number of female customers and male customers entering a store), and we do not care to distinguish between the two types of rare events. Can the combined stream of events be modeled as a Poisson distribution?
Mathematically, let X and Y be independent Poisson random variables with means λ and μ respectively. Prove that $X + Y \sim \text{Pois}(\lambda + \mu)$. (This is known as **Poisson merging**.) Note that it is **not** sufficient to use linearity of expectation to say that $X + Y$ has mean $\lambda + \mu$. You are asked to prove that the *distribution* of $X + Y$ is Poisson.
Note: This property will be extensively used when we discuss Poisson processes.

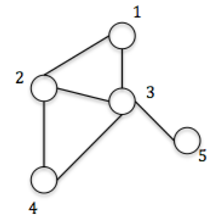

Figure 1: Friendship and clustering coefficient.

Problem 4. **[Extra] Clustering Coefficient**
This problem will explore an important probabilistic concept of clustering that is widely used in machine learning applications today. Consider n students, where n is a positive integer. For each pair of students $i, j \in \{1, \dots, n\}$, $i \neq j$, they are friends with probability p, independently of other pairs. We assume that friendship is mutual. We can see that the friendship among the n students can be represented by an undirected graph G. Let $N(i)$ be the number of friends of student i and $T(i)$ be the number of triangles attached to student i. We define the **clustering coefficient** $C(i)$ for student i as follows:

$$C(i) = \frac{T(i)}{\binom{N(i)}{2}}.$$

The clustering coefficient is not defined for the students who have no friends. An example is shown in Figure 1. Student 3 has 4 friends (1, 2, 4, 5) and there are two triangles attached to student 3, i.e., triangle 1-2-3 and triangle 2-3-4. Therefore $C(3) = 2/\binom{4}{2} = 1/3$.
Find $\mathbb{E}[C(i) \mid N(i) \geq 2]$.