UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session III
Fall 2018

Problem 1. **Change of Variables**

1. Suppose that X has the **standard normal distribution**, that is, X is a continuous random variable with density function
$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{x^2}{2}\Big).$$
What is the density function of $\exp X$? (The answer is called the **lognormal distribution**.)

2. Suppose that X is an arbitrary continuous random variable with density f. What is the density of X^2?

3. What is the answer to the previous question when X has the standard normal distribution? (This is known as the **chi-squared distribution**.)

Problem 2. **Exponential Fun**

1. Let X_1 and X_2 be i.i.d. exponential random variables with parameter λ. Show that the PDF of $X_1 + X_2$ is given by
$$f_{X_1+X_2}(x) = \lambda^2 x e^{-\lambda x}, \text{ for } x > 0.$$

2. Now, for a positive integer n, let $X_1, \dots, X_n$ be i.i.d. exponential random variables with parameter λ and $S_n \triangleq X_1 + \ldots + X_n$. The PDF of S_n is given by the n-fold convolution of the exponential distribution with itself. Show that the PDF of S_n is given by
$$f_{S_n}(x) = \frac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!}, \text{ for } x > 0.$$

3. Using the above result, consider now the random sum $S_N = X_1 + \cdots + X_N$, where N is a geometric random variable with parameter p. Compute the PDF of S_N.

Problem 3. **Order Statistics**
For n a positive integer, let $X_1, \dots, X_n$ be i.i.d. continuous random variables with common PDF f and CDF F. For $i = 1, \dots, n$, let $X^{(i)}$ be the ith smallest of $X_1, \dots, X_n$, so we have $X^{(1)} \le \cdots \le X^{(n)}$. $X^{(i)}$ is known as the i**th order statistic**.

1. What is the CDF of $X^{(i)}$?

2. Differentiate the CDF to obtain the PDF of $X^{(i)}$.

3. Can you obtain the PDF of $X^{(i)}$ directly?