UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session 5
Fall 2018

Problem 1. **Breaking a Stick**
I break a stick n times, where n is a positive integer, in the following manner: the ith time I break the stick, I keep a fraction X_i of the remaining stick where X_i is uniform on the interval $[0, 1]$ and $X_1, X_2, \ldots, X_n$ are i.i.d. Let $P_n = \prod_{i=1}^{n} X_i$ be the fraction of the original stick that I end up with.

1. Show that $P_n^{1/n}$ converges almost surely to some constant function.
2. Compute $\mathbb{E}[P_n]^{1/n}$.

Problem 2. **Mean Square Convergence**
A sequnce of random variables $\{X_n\}_{n\geq 0}$, each satisfying $\mathbb{E}[X_n^2] < \infty$, is said to converge in *mean square* to a random variable X if

$$\lim_{n\to\infty} \mathbb{E}[(X_n - X)^2] = 0.$$

1. Show that convergence in mean square implies convergence in probability.
2. Consider the sequence of random variables $X_{n n\geq 1}$, where each $X_n \sim \mathit{Bernoulli}(1/n)$. Show that this sequence converges to 0 in mean square.
3. Does it converge almost surely?

Problem 3. **Convergence in Probability**
Let $(X_n)_{n=1}^{\infty}$, be a sequence of i.i.d. random variables distributed uniformly in $[-1, 1]$. Show that the following sequences $(Y_n)_{n=1}^{\infty}$ converge in probability to some limit.

(a) $Y_n = (X_n)^n$.

(b) $Y_n = \prod_{i=1}^{n} X_i$.

(c) $Y_n = \max\{X_1, X_2, \ldots, X_n\}$.

(d) $Y_n = (X_1^2 + \cdots + X_n^2)/n$.