UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session 6
Fall 2018

Problem 1. **Markov Chain Practice**
Consider a Markov chain with three states 0, 1, and 2. The transition probabilities are $P(0,1) = P(0,2) = 1/2$, $P(1,0) = P(1,1) = 1/2$, and $P(2,0) = 2/3$, $P(2,2) = 1/3$.

(a) Classify the states in the chain. Is this chain periodic or aperiodic?

(b) In the long run, what fraction of time does the chain spend in state 1?

(c) Suppose that X_0 is chosen according to the steady state distribution. What is $\Pr(X_0 = 0 \mid X_2 = 2)$?

(d) Suppose that $X_0 = 0$, and let T denote the first time by which the process has visited all the states. Find $\mathbb{E}[T]$.

Problem 2. **Running Sum of a Markov Chain**
Let $(X_n)_{n\in\mathbb{N}}$ be a Markov chain with two states, -1 and 1, and transition probabilities $P(-1,1) = P(1,-1) = a$ for $a \in (0,1)$. Define

$$Y_n = X_0 + X_1 + \cdots + X_n.$$

For what values of a is $(Y_n)_{n\in\mathbb{N}}$ a Markov chain?

Problem 3. **Finite Random Walk**

(a) Find the steady-state probabilities $\pi_0, \ldots, \pi_{k-1}$ for the Markov chain in Figure 1. Here, k is a positive integer and $p \in (0,1)$. Express your answer in terms of the ratio $\rho = p/q$, where $q = 1 - p$. Pay particular attention to the special case $\rho = 1$.

(b) Find the limit of π_0 as k approaches infinity; give separate answers for $\rho < 1$, $\rho = 1$, and $\rho > 1$. Find limiting values of π_{k-1} for the same cases.

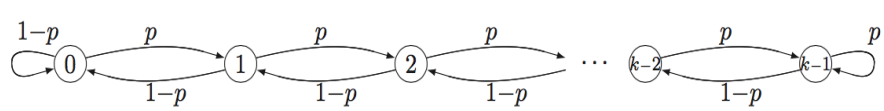

Figure 1: Markov chain for Problem 3