UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Section 7
Fall 2018

Problem 1. **Ant**
An ant is walking on the non-negative integers. At each step, the ant moves forward one step with probability $p \in (0,1)$, or slides back down to 0 with probability $1-p$. What is the average time it takes for the ant to get to n, where n is a positive integer, starting from state 0?

Problem 2. **Before Absorption**
Consider the Markov chain in Figure **??**. Suppose that $X(0) = 1$. Calculate the expected number of times that the chain is in state 1 before being absorbed in state 3. ($X(0) = 1$ is included in this number.)

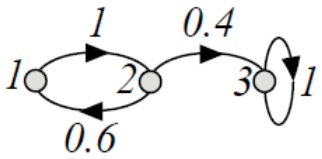

Figure 1: A Markov chain.

Problem 3. **Entropy of a Sum**
Let X_1, X_2 be i.i.d. $Bernoulli(1/2)$ (fair coin flips). Calculate $H(X_1+X_2)$ and show that $H(X_1 + X_2) \geq H(X_1)$. In fact it is generally true that adding independent random variables increases entropy.
Note: It is known that the Gaussian distribution maximizes entropy given a constraint on the variance. Therefore, one intuitive interpretation of the CLT is that convolving independent random variables tends to increase uncertainty until the sum approaches the distribution which "maximizes uncertainty", the Gaussian distribution. Proving the CLT along these lines is far from easy, however.