UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 12
Fall 2018

Issued: Thursday, November 8, 2018 **Due:** Wednesday, November 14, 2018

Problem 1. **Midterm**
Solve all of the problems on the midterm again (including the ones you got correct).

Problem 2. **Gaussian Hypothesis Testing**
Consider a hypothesis testing problem that if $X = 0$, you observe a sample of $\mathcal{N}(\mu_0, \sigma^2)$, and if $X = 1$, you observe a sample of $\mathcal{N}(\mu_1, \sigma^2)$, where $\mu_0, \mu_1 \in \mathbb{R}$, $\sigma^2 > 0$. Find the Neyman-Pearson test for false alarm $\alpha \in (0, 1)$, that is, $\Pr(\hat{X} = 1 \mid X = 0) \leq \alpha$.

Problem 3. **Flipping Coins and Hypothesizing**
You flip a coin until you see heads. Let $X = 0$ be the hypothesis that the bias of the coin (the probability of heads) is p, and $X = 1$ be the hypothesis that the bias of the coin is q, for $q > p$. Solve the hypothesis testing problem: maximize $\Pr[\hat{X} = 1 \mid X = 1]$ subject to $\Pr[\hat{X} = 1 \mid X = 0] \leq \beta$ for $\beta \in [0, 1]$.

Problem 4. **Photodetector LLSE**
Consider a photodetector in an optical communications system that counts the number of photons arriving during a certain interval. A user conveys information by switching a photon transmitter on or off. Assume that the probability of the transmitter being on is p. If the transmitter is on, the number of photons transmitted over the interval of interest is a Poisson random variable Θ with mean λ, and if it is off, the number of photons transmitted is 0. Unfortunately, regardless of whether or not the transmitter is on or off, photons may be detected due to "shot noise". The number N of detected shot noise photons is a Poisson random variable N with mean μ, independent of the transmitted photons. Given the number of detected photons, find the LLSE of the number of transmitted photons.

Problem 5. **Exponential MLE, MAP, Hypothesis Testing**
The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X.

(a) Find $\text{MLE}[X \mid Y]$;

(b) Find $\text{MAP}[X \mid Y]$;

(c) Solve the following hypothesis testing problem:

$$\text{Maximize } \Pr(\hat{X} = 1 \mid X = 1) \text{ subject to } \Pr(\hat{X} = 1 \mid X = a) \leq 5\%$$

where $a > 1$ is given.

Problem 6. **Exam Difficulties**
The difficulty of an EE 126 exam, Θ, is uniformly distributed on $[0, 100]$, and Alice gets a score X that is uniformly distributed on $[0, \Theta]$. Alice gets her score back and wants to estimate the difficulty of the exam.

(a) What is the LLSE for Θ?

(b) What is the MAP of Θ?

Problem 7. Please fill in the survey on Piazza!