Problem 1. Which of the following is always true about two events A and B in a probability space? Select all that apply.

1. $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$.
2. $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$.
3. $\mathbb{P}(A \setminus B) \leq \mathbb{P}(B)$.
4. $\mathbb{P}(A^c) \leq \mathbb{P}(A)$.

Problem 2. Assume n persons arrive at a theater with exactly n seats numbered $1, \ldots, n$. Each person has a ticket with a seat number; however, they ignore the tickets and choose their seats uniformly at random. What is the expected number of persons whose seat numbers is strictly less than their ticket number?

1. $\frac{n-1}{2}$
2. $\frac{n+1}{2}$
3. 1
4. 1/2

Problem 3. Let f and g be 2 probability density functions. Select all that apply:

1. fg is a valid density for any f and g.
2. f^2 is a valid density for all f.
3. $\frac{f}{4} + \frac{3g}{4}$ is a valid density for all f and g.
4. $f - g$ is a valid density for all f and g.

Problem 4. Let X and Y be independent Bernoulli random variables with parameters $\frac{1}{2}$. What can you say about the random variables $X + Y$ and $|X - Y|$? (Select **all** that apply)

1. Independent
2. Uncorrelated
3. Correlated
4. Dependent

Problem 5. From a set of cards numbered $1, \ldots, 10$, we draw 3 cards without replacement uniformly at random, i.e. the first card is chosen uniformly at random from the 10 cards, the second card is chosen uniformly at random from the remaining 9 cards, and the third card is chosen uniformly at random from the remaining 8 cards. What is the expectation of the sum of the numbers on these 3 cards?

1. 11
2. 16.5
3. 19.5
4. 12

Problem 6. A communication network has n inputs and m outputs. Assume there are X_i many packets arriving at a given input $1 \leq i \leq n$, where X_i's are independent and each one is distributed according to a Poisson with parameter $\lambda > 0$. Furthermore, assume that each packet is independently routed to one of the m outputs uniformly at random. What is the variance of the packets arriving at the first output?

1. $\lambda n/m$
2. $\lambda m/n$
3. $m(\lambda/n)^2$
4. $n(\lambda/m)^2$

Problem 7. With what value of constant C does $p_m := C2^m/m!$ for $m = 1, 2, \ldots$ become a valid probability mass function?

1. e^2
2. e^{-2}
3. $\frac{1}{e^2-1}$
4. $\frac{1}{e^{-2}-1}$.

Problem 8. Let X and Y be positive independent continuous random variables and U be independent of X and Y. U takes value $\{-1, +1\}$ with probability $\frac{1}{2}$ each. Define, $S = UX$, and $T = UY$. Choose the correct answers (Select **all** that apply):

1. S and T are independent
2. S and T are dependent
3. S^2 and T^2 are uncorrelated
4. S^2 and T^2 are correlated

Diagnostic Quiz Answer Sheet

Name: ____________________ SID: ____________________

Q1

Q2

Q3

Q4

Q5

Q6

Q7

Q8

Useful formulas:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sum_{r=0}^{\infty} ar^n = \frac{a}{1-r}, |r| < 1$$