Problem 1. Which of the following is always true about two events A and B in a probability space? Select all that apply.

- 1. $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B).$
- 2. $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$.
- 3. $\mathbb{P}(A \setminus B) \leq \mathbb{P}(B)$.
- 4. $\mathbb{P}(A^c) \leq \mathbb{P}(A)$.

Problem 2. Assume n persons arrive at a theater with exactly n seats numbered $1, \ldots, n$. Each person has a ticket with a seat number; however, they ignore the tickets and choose their seats uniformly at random. What is the expected number of persons whose seat numbers is strictly less than their ticket number?

1. $\frac{n-1}{2}$ 2. $\frac{n+1}{2}$ 3. 1 4. 1/2

Problem 3. Let f and g be 2 probability density functions. Select all that apply:

1. fg is a valid density for any f and g.

- 2. f^2 is a valid density for all f.
- 3. $\frac{f}{4} + \frac{3g}{4}$ is a valid density for all f and g.
- 4. f g is a valid density for all f and g.

Problem 4. Let X and Y be independent Bernoulli random variables with parameters $\frac{1}{2}$. What can you say about the random variables X + Y and |X - Y|? (Select all that apply)

- 1. Independent
- 2. Uncorrelated
- 3. Correlated
- 4. Dependent

Problem 5. From a set of cards numbered $1, \ldots, 10$, we draw 3 cards without replacement uniformly at random, i.e. the first card is chosen uniformly at random from the 10 cards, the second card is chosen uniformly at random from the remaining 9 cards, and the third card is chosen uniformly at random from the remaining 8 cards. What is the expectation of the sum of the numbers on these 3 cards?

- $1. \ 11$
- $2.\ 16.5$
- 3. 19.5
- 4. 12

Problem 6. A communication network has n inputs and m outputs. Assume there are X_i many packets arriving at a given input $1 \leq i \leq n$, where X_i 's are independent and each one is distributed according to a Poisson with parameter $\lambda > 0$. Furthermore, assume that each packet is independently routed to one of the m outputs uniformly at random. What is the variance of the packets arriving at the first output?

- 1. $\lambda n/m$
- 2. $\lambda m/n$ 3. $m(\lambda/n)^2$
- 4. $n(\lambda/m)^2$

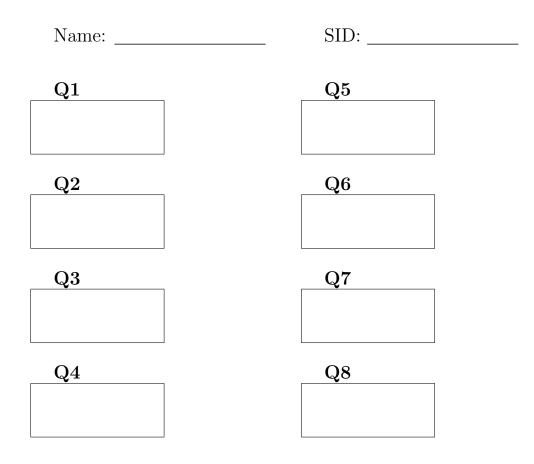
Problem 7. With what value of constant C does $p_m := C2^m/m!$ for m = 1, 2, ... become a valid probability mass function?

1. e^2 2. e^{-2} 3. $\frac{1}{e^2-1}$ 4. $\frac{1}{e^{-2}-1}$.

Problem 8. Let X and Y be positive independent continuous random variables and U be independent of X and Y. U takes value $\{-1, +1\}$ with probability $\frac{1}{2}$ each. Define, S = UX, and T = UY. Choose the correct answers (Select **all** that apply):

- 1. S and T are independent
- 2. S and T are dependent
- 3. S^2 and T^2 are uncorrelated
- 4. S^2 and T^2 are correlated

Diagnostic Quiz Answer Sheet



<u>Useful formulas:</u>

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sum_{r=0}^{\infty}ar^n=\frac{a}{1-r}, |r|<1$$