UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 11
Fall 2019

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1. **Gaussians and the MSE**

   Suppose you draw n i.i.d. data points $(x_1, y_1), \ldots, (x_n, y_n)$, where n is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, Y has a linear dependence on X, with additive Gaussian noise.) Show that finding the MLE estimate of W given the data points $\{(x_i, y_i) : i = 1, \ldots, n\}$ is equivalent to minimizing the cost function

$$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2$$

2. **Hypothesis Testing for Bernoulli Random Variables**

   Assume that

   - If $X = 0$, then $Y \sim \text{Bernoulli}(1/4)$.
   - If $X = 1$, then $Y \sim \text{Bernoulli}(3/4)$.

   Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $r : \{0, 1\} \to \{0, 1\}$ with respect to the criterion

$$\min_{\text{randomized } r:\{0,1\}\to\{0,1\}} \mathbb{P}\big(r(Y) = 0 \;\big|\; X = 1\big)$$
$$\text{s.t. } \mathbb{P}\big(r(Y) = 1 \;\big|\; X = 0\big) \leq \beta,$$

   where $\beta \in [0, 1]$ is a given upper bound on the false positive probability.

3. **Bayesian Hypothesis Testing for Gaussian Distribution**

   Assume that X has prior probabilities $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = 1/2$. Further

   - If $X = 0$, then $Y \sim \mathcal{N}(\mu_0, \sigma_0^2)$.
   - If $X = 1$, then $Y \sim \mathcal{N}(\mu_1, \sigma_1^2)$.

   Assume $\mu_0 < \mu_1$ and $\sigma_0 < \sigma_1$.

   Using the Bayesian formulation of hypothesis testing, find the optimal *decision rule* $r : \mathbb{R} \to \{0, 1\}$ with respect to the minimum expected cost criterion

$$\min_{r:\mathbb{R}\to\{0,1\}} \mathbb{E}[I\{r(Y) \neq X\}].$$