UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 3
Fall 2019

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1. **Triangle Density**

   Consider random variables X and Y which have a joint PDF uniform on the triangle with vertices at $(0,0),(1,0),(0,1)$.

   (a) Find the joint PDF of X and Y.

   (b) Find the marginal PDF of Y.

   (c) Find the conditional PDF of X given Y.

   (d) Find $\mathbb{E}[X]$ in terms of $\mathbb{E}[Y]$.

   (e) Find $\mathbb{E}[X]$.

2. **Change of Variables**

   Let X be a R.V with PDF $f_X(x)$ and CDF $F_X(x)$. Let $g(X)$ be an invertible function. The change of variables problem asks for the density of $Y = g(X)$ which is a new R.V. To find this distribution, we use the definition of the CDF

   $$F_Y(y) = P(Y \le y) = P(g(X) \le y) = P(X \le g^{-1}(y)) = F_X(g^{-1}(y))$$

   How would you do this if the function g was not invertible? For example $g(x) = x^2 \forall x \in \mathbb{R}$ has two values in the domain mapping to each value in its range. In these cases we have to include all parts of the domain that contribute to the probability. In the case of a discrete distribution using the above g, this would look like

   $$P(Y = y) = P(g(X) = y) = P(X \in \{-\sqrt{y}, \sqrt{y}\})$$

   (a) Suppose that X has the **standard normal distribution**, that is, X is a continuous random variable with density function

   $$f(x) = \frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{x^2}{2}\Big).$$

   What is the density function of $\exp X$? (The answer is called the **lognormal distribution**.)

   (b) Suppose that X is a continuous random variable with density f. What is the density of X^2?

   (c) What is the answer to the previous question when X has the standard normal distribution? (This is known as the **chi-squared distribution**.)

3. Order Statistics

For n a positive integer, let $X_1, \ldots, X_n$ be i.i.d. continuous random variables with common PDF f and CDF F. For $i = 1, \ldots, n$, let $X^{(i)}$ be the ith smallest of $X_1, \ldots, X_n$, so we have $X^{(1)} \leq \cdots \leq X^{(n)}$. $X^{(i)}$ is known as the **ith order statistic**.

(a) What is the CDF of $X^{(i)}$?

(b) Differentiate the CDF to obtain the PDF of $X^{(i)}$.

(c) Can you obtain the PDF of $X^{(i)}$ directly?