UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 9
Fall 2019

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1. **Continuous-Time Markov Chains: Introduction**

Consider the continuous-time Markov process with state space $\{1, 2, 3, 4\}$ and the rate matrix

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

(a) Find the stationary distribution p of the Markov process.

(b) Find the stationary distribtion π of the jump chain, i.e., the discrete-time Markov chain which only keeps track of the jumps of the CTMC. Formally, if the CTMC $(X(t))_{t\geq 0}$ jumps at times $T_1, T_2, T_3, \ldots$, then the DTMC is defined as $(Y_n)_{n=1}^{\infty}$ where $Y_n := X_{T_n}$.

(c) Suppose the chain starts in state 1. What is the expected amount of time until it changes state for the first time?

(d) Again assume the chain starts in state 1. What is the expected amount of time until the chain is in state 4?

2. $M/M/2$ **Queue**

A queue has Poisson arrivals with rate λ. It has two servers that work in parallel. When there are at least two customers in the queue, two are being served. When there is only one customer, only one server is active. The service times are i.i.d. exponential random variables with rate μ. Let $X(t)$ be the number of customers either in the queue or in service at time t.

(a) Argue that the process $(X(t), t \geq 0)$ is a Markov process.

(b) Draw the state transition diagram.

(c) Find the range of values of μ for which the Markov chain is positive-recurrent and for this range of values calculate the stationary distribution of the Markov chain.

3. **Machine**

A machine, once in production mode, operates continuously until an alarm signal is generated. The time up to the alarm signal is an exponential random variable with parameter 1. Subsequent to the alarm signal, the machine is tested for an exponentially distributed amount of time with parameter 5. The test results are positive, with probability 1/2, in which case the machine

returns to production mode, or negative, with probability 1/2, in which case the machine is taken for repair. The duration of the repair is exponentially distributed with parameter 3.

(a) Let states $1, 2, 3$ correspond to production mode, testing, and repair, respectively. Let $(X(t))_{t\geq 0}$ denote the state of the system at time t. Is $(X(t))_{t\geq 0}$ a CTMC?

(b) Find the rate and transition matrices.

(c) Find the steady state probabilities.