UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Problem Set 5
Fall 2019

Issued: September 26th, 2019 **Due:** 11:59 PM, October 2nd, 2019

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1. **Midterm**

   Solve all of the problems on the midterm again (including the ones you got correct). The midterm will be released by Thursday night (09/26) on the website.

2. **Really Random Binomial**

   You have a binomial random variable $X \sim \text{Bin}(n, u)$. Unfortunately, you lost information about what the value u is, so you assume that u is now a random variable $U \sim \text{Unif}[0, 1]$, since you know it is a probability. Given that you sample from this binomial distribution and observe k successes, find the posterior distribution of U.

   Hint: Use MGFs to compute $\mathbb{P}(X = k)$ instead of integrating the distribution directly. The binomial theorm might also be useful here. Finally, recall the identity $\sum_{i=0}^{n} s^i = \frac{1-s^{n+1}}{(1-s)}$.

3. **Poisson Bounds**

   Let X be the sum of 20 i.i.d. Poisson random variables $X_1, \ldots, X_{20}$ with $\mathbb{E}[X_1] = 1$. Use the following techniques to upper bound $\mathbb{P}(X \geq 26)$.

   (a) Markov's Inequality

   (b) Chebyshev's Inequality

   (c) Chernoff Bound

4. **Tricky Markov Bound**

   Suppose $\mathbb{E}[X] = 0$, $\text{var}(X) = \sigma^2 < \infty$, and $\alpha > 0$. Prove the following bound:

   $$\mathbb{P}(X \geq \alpha) \leq \frac{\sigma^2}{\alpha^2 + \sigma^2}.$$

5. **Confidence Interval Comparisons**

   In order to estimate the probability of a head in a coin flip, p, you flip a coin n times, where n is a positive integer, and count the number of heads, S_n. You use the estimator $\hat{p} = S_n/n$.

   (a) You choose the sample size n to have a guarantee

   $$\mathbb{P}(|\hat{p} - p| \geq \epsilon) \leq \delta.$$

   Using Chebyshev Inequality, determine n with the following parameters. Note that you should not have p in your final answer.

(i) Compare the value of n when $\epsilon = 0.05$, $\delta = 0.1$ to the value of n when $\epsilon = 0.1, \delta = 0.1$.

(ii) Compare the value of n when $\epsilon = 0.1, \delta = 0.05$ to the value of n when $\epsilon = 0.1, \delta = 0.1$.

(b) Now, we change the scenario slightly. You know that $p \in (0.4, 0.6)$ and would now like to determine the smallest n such that

$$\mathbb{P}\Big(\frac{|\hat{p} - p|}{p} \leq 0.05\Big) \geq 0.95.$$

Use the CLT to find the value of n that you should use. *Recall that the CLT states that the sum of IID random variables tends to a normal distribution with the sample mean and variance as it's parameters for n large enough.*