UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 1
Fall 2020

1. Miscellaneous Review

(a) Show that the probability that exactly one of the events A and B occurs is $\mathbb{P}(A)+\mathbb{P}(B)-2\mathbb{P}(A\cap B)$.

(b) If A is independent of itself, show that $\mathbb{P}(A) = 0$ or 1.

2. Joint Occurrence

You know that, at least one of the events A_r (for $r \in \{1, \dots, n\}$, where n is an integer ≥ 2) is certain to occur but certainly no more than two occur. Show that if the probability of occurrence of any single event is p, and the probability of joint occurrence of any two distinct events is q, we have $p \geq 1/n$ and $q \leq 2/[n(n-1)]$.

3. Colored Sphere

Consider a sphere that has $\frac{1}{10}$ of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

Hint: Carefully define some relevant events.

4. [Bonus] Borel-Cantelli Lemma

Prove the **Borel-Cantelli Lemma**: If $A_1, A_2, \dots$ is a sequence of events with $\sum_{i=1}^{\infty} \mathbb{P}(A_i) < \infty$, then

$$\mathbb{P}(\text{infinitely many of } A_1, A_2, \dots \text{ occur}) = 0.$$