1. Miscellaneous Review
   
   (a) Show that the probability that exactly one of the events $A$ and $B$ occurs is $P(A) + P(B) - 2P(A \cap B)$.
   
   (b) If $A$ is independent of itself, show that $P(A) = 0$ or $1$.

2. Joint Occurrence
   
   You know that, at least one of the events $A_r$ (for $r \in \{1, \ldots, n\}$, where $n$ is an integer $\geq 2$) is certain to occur but certainly no more than two occur. Show that if the probability of occurrence of any single event is $p$, and the probability of joint occurrence of any two distinct events is $q$, we have $p \geq 1/n$ and $q \leq 2/[n(n - 1)]$.

3. Colored Sphere
   
   Consider a sphere that has $\frac{1}{10}$ of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

   *Hint: Carefully define some relevant events.*

4. [Bonus] Borel-Cantelli Lemma
   
   Prove the Borel-Cantelli Lemma: If $A_1, A_2, \ldots$ is a sequence of events with $\sum_{i=1}^{\infty} P(A_i) < \infty$, then
   
   $$P(\text{infinitely many of } A_1, A_2, \ldots \text{ occur}) = 0.$$