UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 10
Fall 2020

1. Gaussians and the MSE

Suppose you draw n i.i.d. data points $(x_1, y_1), \dots, (x_n, y_n)$, where n is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, Y has a linear dependence on X, with additive Gaussian noise.) Show that finding the MLE estimate of W given the data points $\{(x_i, y_i) : i = 1, \dots, n\}$ is equivalent to minimizing the cost function

$$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2$$

2. Hypothesis Testing for Bernoulli Random Variables

Assume that

- If $X = 0$, then $Y \sim \text{Bernoulli}(1/4)$.
- If $X = 1$, then $Y \sim \text{Bernoulli}(3/4)$.

Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $r : \{0, 1\} \to \{0, 1\}$ with respect to the criterion

$$\min_{\text{randomized } r:\{0,1\}\to\{0,1\}} \mathbb{P}\big(r(Y) = 0 \bigm| X = 1\big)$$
$$\text{s.t. } \mathbb{P}\big(r(Y) = 1 \bigm| X = 0\big) \le \beta,$$

where $\beta \in [0, 1]$ is a given upper bound on the false positive probability.

3. Bayesian Hypothesis Testing for Gaussian Distribution

Assume that X has prior probabilities $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = 1/2$. Further

- If $X = 0$, then $Y \sim \mathcal{N}(\mu_0, \sigma_0^2)$.
- If $X = 1$, then $Y \sim \mathcal{N}(\mu_1, \sigma_1^2)$.

Assume $\mu_0 < \mu_1$ and $\sigma_0 < \sigma_1$.

Using the Bayesian formulation of hypothesis testing, find the optimal *decision rule* $r : \mathbb{R} \to \{0, 1\}$ with respect to the minimum expected cost criterion

$$\min_{r:\mathbb{R}\to\{0,1\}} \mathbb{E}[I\{r(Y) \ne X\}].$$