

**Discussion 4**

Spring 2020

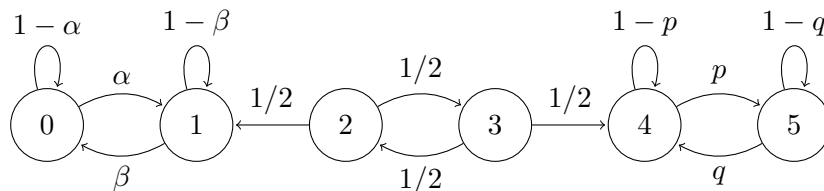
**1. Hitting Time with Coins**

Let's consider a sequence of fair coin flips.

- (a) What's the expected number of coin flips until we first see two heads in a row?
- (b) What's the expected number of coin flips until we see a head and then immediately a tail?

**2. Reducible Markov Chain**

Consider the following Markov chain, for  $\alpha, \beta, p, q \in (0, 1)$ .



- (a) Find all the recurrent and transient classes.
- (b) Given that we start in state 2, what is the probability that we will reach state 0 before state 5?
- (c) What are all of the possible stationary distributions of this chain?
- (d) Suppose we start in the initial distribution  $\pi_0 := [0 \ 0 \ \gamma \ 1 - \gamma \ 0 \ 0]$  for some  $\gamma \in [0, 1]$ . Does the distribution of the chain converge, and if so, to what?

**3. Random Walk on an Undirected Graph**

Consider a random walk on an undirected connected finite graph (that is, define a Markov chain where the state space is the set of vertices of the graph, and at each time step, transition to a vertex chosen uniformly at random out of the neighborhood of the current vertex). What is the stationary distribution?