

Discussion 5

Fall 2020

1. Convergence of Exponentials

Let X_1, X_2, \dots be i.i.d. $\text{Exponential}(\lambda)$ random variables. Show that

$$\frac{X_n}{\ln n} \rightarrow 0 \quad \text{in probability as } n \rightarrow \infty.$$

2. Breaking a Stick

I break a stick n times, where n is a positive integer, in the following manner: the i th time I break the stick, I keep a fraction X_i of the remaining stick where X_i is uniform on the interval $[0, 1]$ and X_1, X_2, \dots, X_n are i.i.d. Let $P_n = \prod_{i=1}^n X_i$ be the fraction of the original stick that I end up with.

- (a) Show that $P_n^{1/n}$ converges almost surely to some constant function. (**Hint:** use the continuous mapping theorem. What continuous map would let use the SLLN?)
- (b) Compute $\mathbb{E}[P_n]^{1/n}$.

3. r^{th} Moment Convergence

Let X_n converge in the r^{th} mean to X . Prove that it converges in probability.