UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 6
Fall 2020

1. Generating Random Variables

Consider a continuous random variable $U \sim \text{Uniform}[0,1]$. Let $F : \mathbb{R} \to [0,1]$ be a strictly increasing distribution function. Show that $F^{-1}(U)$ has the cumulative distribution function (CDF) F.

2. Gaussian Tail Bounds

Let $\phi(y) = \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}}$ be the PDF of a standard normal random variable $Y \sim \mathcal{N}(0,1)$.

(a) Show that for $y \neq 0$ we have that

$$\phi(y) = -\frac{1}{y} \cdot \phi'(y).$$

(b) Use (a) to show that

$$\mathbb{P}(Y \geq t) \leq \frac{1}{t} \cdot \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}}, \quad \text{for all } t > 0.$$

3. Revisiting Facts Using Transforms

(a) Let $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\mu)$ be independent. Calculate the characteristic function of $X + Y$ and use this to show that $X + Y \sim \text{Poisson}(\lambda + \mu)$.

(b) Calculate the characteristic function of $X \sim \text{Exponential}(\lambda)$ and use this to find all of the moments of X.

(c) Repeat the above part, but for $X \sim \mathcal{N}(0,1)$.