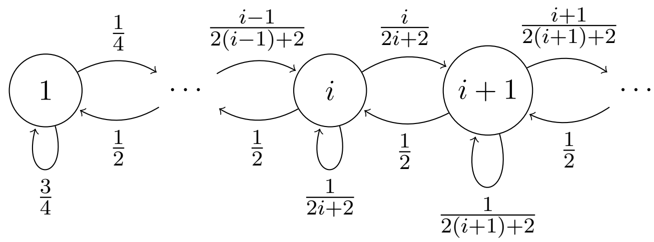
UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 7
Fall 2020

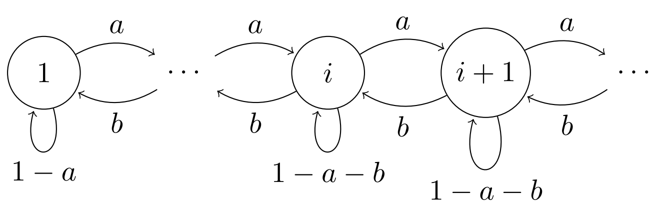
1. Markov Chains with Countably Infinite State Space

(a) Consider a Markov chain with state space $\mathbb{Z}_{>0}$ and transition probability graph:



Show that this Markov chain has no stationary distribution.

(b) Consider a Markov chain with state space $\mathbb{Z}_{>0}$ and transition probability graph:



Assume that $0 < a < b$ and $0 < a + b \leq 1$. Show that the probability distribution given by

$$\pi(i) = \left(\frac{a}{b}\right)^{i-1}\left(1 - \frac{a}{b}\right), \text{ for } i \in \mathbb{Z}_{>0},$$

is a stationary distribution of this Markov chain.

2. Poisson Practice

Let $(N(t), t \geq 0)$ be a Poisson process with rate λ. Let T_k denote the time of k-th arrival, for $k \in \mathbb{N}$, and given $0 \leq s < t$, we write $N(s, t) = N(t) - N(s)$. Compute:

(a) $\mathbb{P}(N(1) + N(2, 4) + N(3, 5) = 0)$.
(b) $\mathbb{E}(N(1, 3) \mid N(1, 2) = 3)$.
(c) $\mathbb{E}(T_2 \mid N(2) = 1)$.

3. Continuous-Time Markov Chains: Introduction

Consider the continuous-time Markov process with state space $\{1, 2, 3, 4\}$ and the rate matrix

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

(a) Find the stationary distribution p of the Markov process.

(b) Find the stationary distribtion π of the jump chain, i.e., the discrete-time Markov chain which only keeps track of the jumps of the CTMC. Formally, if the CTMC $(X(t))_{t\geq 0}$ jumps at times $T_1, T_2, T_3, \ldots$, then the DTMC is defined as $(Y_n)_{n=1}^{\infty}$ where $Y_n := X_{T_n}$.

(c) Suppose the chain starts in state 1. What is the expected amount of time until it changes state for the first time?

(d) Again assume the chain starts in state 1. What is the expected amount of time until the chain is in state 4?