UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 8

Fall 2020

1. System Shocks

For a positive integer n, let $X_1, \ldots, X_n$ be independent exponentially distributed random variables, each with mean 1. Let $\gamma > 0$.

A system experiences shocks at times $k = 1, \ldots, n$. The size of the shock at time k is X_k.

(a) Suppose that the system fails if any shock exceeds the value γ. What is the probability of system failure?

(b) Suppose instead that the effect of the shocks is cumulative, i.e., the system fails when the total amount of shock received exceeds γ. What is the probability of system failure?

2. Spatial Poisson Process

A two-dimensional Poisson process of rate $\lambda > 0$ is a process of randomly occurring special points in the plane such that (i) for any region of area A the number of special points in that region has a Poisson distribution with mean λA, and (ii) the number of special points in non-overlapping regions is independent. For such a process consider an arbitrary location in the plane and let X denote its distance from its nearest special point (where distance between two points (x_1, y_1) and (x_2, y_2) is defined as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$). Show that:

(a) $\mathbb{P}(X > t) = \exp(-\lambda \pi t^2)$ for $t > 0$.

(b) $\mathbb{E}[X] = \dfrac{1}{2\sqrt{\lambda}}$.

3. M-M-∞ Queues

A continuous-time queue has Poisson arrivals with rate λ, and it is equipped with infinitely many servers. The servers can work in parallel on multiple customers, but they are non-cooperative in the sense that a single customer can only be served by one server. Thus, when there are k customers in the queue ($k \in \mathbb{N}$), k servers are active. Suppose that the service time of each customer is exponentially distributed with rate μ and they are i.i.d.

(a) Argue that the queue-length is a Markov chain. Draw the transition diagram of the Markov chain.

(b) Prove that for all finite values of λ and μ the Markov chain is positive-recurrent and find the invariant distribution.