UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 9
Fall 2020

1. Expected Squared Arrival Times

Let $(N(t), t \geq 0)$ be a Poisson process with arrival instants $(T_n, n \in \mathbb{N})$, where $0 < T_1 < T_2 < \cdots$. Find $\mathbb{E}(\sum_{k=1}^{3} T_k^2 \mid N(1) = 3)$.

2. Bounds on Entropy

There's actually a limit to how much "randomness" there is in a random variable X that takes on $|\mathcal{X}|$ distinct values. Prove that for any distribution p_X, $H(X) \leq \log |\mathcal{X}|$.

Hint: We can write $H(X)$ as $\mathbb{E}[\log \frac{1}{P_X(X)}]$. By Jensen's inequality, if we have a random variable Z and a concave function f, $f(\mathbb{E}[Z]) \geq \mathbb{E}[f(Z)]$.

3. Exponential: MLE & MAP

The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X.

(a) Find MLE$[X \mid Y]$.

(b) Find MAP$[X \mid Y]$.