UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Problem Set 12
Fall 2020

1. Midterm

Solve all of the problems on the midterm again (including the ones you got correct).

2. Gaussian LLSE

The random variables X, Y, Z are i.i.d. $\mathcal{N}(0,1)$.

(a) Find $L[X^2 + Y^2 \mid X + Y]$.

(b) Find $L[X + 2Y \mid X + 3Y + 4Z]$.

(c) Find $L[(X + Y)^2 \mid X - Y]$.

3. Photodetector LLSE

Consider a photodetector in an optical communications system that counts the number of photons arriving during a certain interval. A user conveys information by switching a photon transmitter on or off. Assume that the probability of the transmitter being on is p. If the transmitter is on, the number of photons transmitted over the interval of interest is a Poisson random variable Θ with mean λ, and if it is off, the number of photons transmitted is 0. Unfortunately, regardless of whether or not the transmitter is on or off, photons may be detected due to "shot noise". The number N of detected shot noise photons is a Poisson random variable N with mean μ, independent of the transmitted photons. Let T be the number of transmitted photons and D be the number of detected photons. Find $L[T \mid D]$.

4. Noisy Guessing

Let X, Y, and Z be i.i.d. with the standard Gaussian distribution. Find $\mathbb{E}[X \mid X + Y, X + Z, Y - Z]$.

Hint: Argue that the observation $Y - Z$ is redundant.

5. Jointly Gaussian Decomposition

Let U and V be jointly Gaussian random variables with means $\mu_U = 1$, $\mu_V = 4$, respectively, with variances $\sigma_U^2 = 2.5$, $\sigma_V^2 = 2$, respectively, and with covariance $\rho = 1$. Can we write U as $U = aV + Z$, where a is a scalar and Z is independent of V? If you think we can, find the value of a and the distribution of Z; otherwise please explain the reason.