UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 11
Fall 2021

1. BSC: MLE & MAP

You are testing a digital link that corresponds to a BSC with some error probability $\epsilon \in [0, 0.5]$.

(a) Assume you observe the input and the output of the link. How do you find the MLE of ϵ?

(b) You are told that the inputs are i.i.d. bits that are equal to 1 with probability 0.6 and to 0 with probability 0.4. You observe n outputs (n is a positive integer). How do you calculate the MLE of ϵ?

(c) The situation is as in the previous case, but you are told that ϵ has PDF $4-8x$ on $[0, 0.5)$. How do you calculate the MAP of ϵ given n outputs? You may leave your answer in terms of quadratic equation to be solved.

2. Hypothesis Testing for Uniform Distribution

Assume that

- If $X = 0$, then $Y \sim \text{Uniform}[-1, 1]$.
- If $X = 1$, then $Y \sim \text{Uniform}[0, 2]$.

Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $r : [-1, 2] \to \{0, 1\}$ with respect to the criterion

$$\min_{\text{randomized } r:[-1,2]\to\{0,1\}} P\big(r(Y) = 0 \mid X = 1\big)$$
$$\text{s.t. } P\big(r(Y) = 1 \mid X = 0\big) \le \beta,$$

where $\beta \in [0, 1]$ is a given upper bound on the false positive probability.

3. Bayesian Hypothesis Testing for Gaussian Distribution

Assume that X has prior probabilities $P(X = 0) = P(X = 1) = 1/2$. Further

- If $X = 0$, then $Y \sim \mathcal{N}(\mu_0, \sigma_0^2)$.
- If $X = 1$, then $Y \sim \mathcal{N}(\mu_1, \sigma_1^2)$.

Assume $\mu_0 < \mu_1$ and $\sigma_0 < \sigma_1$.

Using the Bayesian formulation of hypothesis testing, find the optimal *decision rule* $r : \mathbb{R} \to \{0, 1\}$ with respect to the minimum expected cost criterion

$$\min_{r:\mathbb{R}\to\{0,1\}} \mathbb{E}[I\{r(Y) \ne X\}].$$