

Discussion 12

Fall 2021

1. Hypothesis Testing for Bernoulli Random Variables

Assume that

- If $X = 0$, then $Y \sim \text{Bernoulli}(1/4)$.
- If $X = 1$, then $Y \sim \text{Bernoulli}(3/4)$.

Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $r : \{0, 1\} \rightarrow \{0, 1\}$ with respect to the criterion

$$\begin{aligned} \min_{\text{randomized } r: \{0,1\} \rightarrow \{0,1\}} & P(r(Y) = 0 \mid X = 1) \\ \text{s.t.} & P(r(Y) = 1 \mid X = 0) \leq \beta, \end{aligned}$$

where $\beta \in [0, 1]$ is a given upper bound on the false positive probability.

2. Joint Gaussian Probability

Let $X \sim \mathcal{N}(1, 1)$ and $Y \sim \mathcal{N}(0, 1)$ be jointly Gaussian with covariance ρ . What is $P(X > Y)$?