UC Berkeley Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 12

Fall 2021

1. Hypothesis Testing for Bernoulli Random Variables

Assume that

- If X = 0, then $Y \sim \text{Bernoulli}(1/4)$.
- If X = 1, then $Y \sim \text{Bernoulli}(3/4)$.

Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized decision rule $r: \{0,1\} \to \{0,1\}$ with respect to the criterion

$$\begin{aligned} \min_{\text{randomized } r: \{0,1\} \to \{0,1\}} P\big(r(Y) = 0 \mid X = 1\big) \\ \text{s.t. } P\big(r(Y) = 1 \mid X = 0\big) \le \beta, \end{aligned}$$

where $\beta \in [0,1]$ is a given upper bound on the false positive probability.

2. Joint Gaussian Probability

Let $X \sim \mathcal{N}(1,1)$ and $Y \sim \mathcal{N}(0,1)$ be jointly Gaussian with covariance ρ . What is P(X > Y)?