UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 8
Fall 2021

1. Merging & Splitting

(a) (Splitting/Thinning) Let N_t be a Poisson process with rate λ. Let $0 < p < 1$, and consider the split processes $N_t^{(1)}$ and $N_t^{(2)}$, where each arrival of N_t is routed to $N_t^{(1)}$ with probability p, and $N_t^{(2)}$ with probability $1-p$. Show that $N_t^{(1)}$ is a Poisson process of rate λp, and $N_t^{(2)}$ a Poisson process of rate $\lambda(1-p)$. Also show that these two processes are independent.

(b) (Merging) Let $N_t^{(1)}$ and $N_t^{(2)}$ be two independent Poisson processes of rates λ and μ. Suppose we merge them, i.e. consider N_t the process denoting total arrivals from both processes. Show that N_t is a Poisson process of rate $\lambda + \mu$.

2. Machine

A machine, once in production mode, operates continuously until an alarm signal is generated. The time up to the alarm signal is an exponential random variable with parameter 1. Subsequent to the alarm signal, the machine is tested for an exponentially distributed amount of time with parameter 5. The test results are positive, with probability 1/2, in which case the machine returns to production mode, or negative, with probability 1/2, in which case the machine is taken for repair. The duration of the repair is exponentially distributed with parameter 3.

(a) Let states $1, 2, 3$ correspond to production mode, testing, and repair, respectively. Let $(X(t))_{t\geq 0}$ denote the state of the system at time t. Is $(X(t))_{t\geq 0}$ a CTMC?

(b) Find the rate matrix Q of the CTMC and the transition matrix P of the corresponding jump chain.

(c) Find the stationary distribution of the CTMC.

3. Lazy Server

Customers arrive at a queue in a service facility at the times of a Poisson process of rate λ. The service facility has infinite capacity. There is an infinitely powerful but lazy server who visits the service facility at the times of a Poisson process of rate μ. These two processes are independent. When the server visits the facility she instantaneously serves all the customers that are in the queue and then immediately leaves (until her next visit).

Thus, for instance, at any time, any customers that are waiting in the queue would only be those that arrived after the most recent visit of the server.

(a) Model the queue length as a CTMC and find the stationary distribution.

(b) Suppose tha the CTMC is at stationary, and find the mean number of customers waiting in the queue at any given time.