UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Problem Set 8
Fall 2021

1. **Geometric Sum of Exponentials**

   Let $X_1, X_2, \ldots$ be iid exponentials with parameter λ. If $N \sim \text{Geom}(p)$ taking values on $\{1, 2, \ldots\}$, then show that
   $$\sum_{i=1}^{N} X_i$$
   is exponential and determine its parameter. *Hint:* Consider Poisson thinning.

2. **Bus Arrivals at Cory Hall**

   Starting at time 0, the 52 line makes stops at Cory Hall according to a Poisson process of rate λ. Students arrive at the stop according to an independent Poisson process of rate μ. Every time the bus arrives, all students waiting get on.

   (a) Given that the interarrival time between bus $i-1$ and bus i is x, find the distribution for the number of students entering the ith bus. (Here, x is a given number, not a random quantity.)

   (b) Given that a bus arrived at 9:30 AM, find the distribution for the number of students that will get on the next bus.

   (c) Find the distribution of the number of students getting on the next bus to arrive after 9:30 AM, assuming that time 0 was infinitely far in the past.

3. **Frogs**

   Three frogs are playing near a pond. When they are in the sun they get too hot and jump in the lake at rate 1. When they are in the lake they get too cold and jump onto the land at rate 2. The rates here refer to the rate in exponential distribution. Let X_t be the number of frogs in the sun at time $t \geq 0$.

   (a) Find the stationary distribution for $(X_t)_{t \geq 0}$.

   (b) Check the answer to (a) by noting that the three frogs are independent two-state Markov chains.

4. **Taxi Queue**

   Empty taxis pass by a street corner according to a Poisson process of rate two per minute and pick up a passenger if one is waiting there. Passengers arrive at the street corner according to a Poisson process of rate one per minute and wait for a taxi only if there are less than four persons waiting; otherwise they leave and never return. John arrives at the street corner at a given time. Find his expected waiting time, given that he joins the queue. Assume that the process is in steady state.

5. $M/M/2$ **Queue**

A queue has Poisson arrivals with rate λ. It has two servers that work in parallel. When there are at least two customers in the queue, two are being served. When there is only one customer, only one server is active. The service times are i.i.d. exponential random variables with rate μ. Let $X(t)$ be the number of customers either in the queue or in service at time t.

(a) Argue that the process $(X(t), t \geq 0)$ is a Markov process.

(b) Draw the state transition diagram.

(c) Find the range of values of μ for which the Markov chain is positive-recurrent and for this range of values calculate the stationary distribution of the Markov chain.