1. Poisson Process Practice

Let \((N_t)_{t \geq 0}\) be a Poisson process with rate \(\lambda\). Let \(T_k, k \in \mathbb{N}\) denote the time of the \(k\)th arrival. Given \(0 \leq s < t\), we write \(N(s, t) := N(t) - N(s)\). Compute the following:

a. \(\mathbb{P}(N(1) + N(2, 4) + N(3, 5) = 0)\).

b. \(\mathbb{E}(N(1, 3) \mid N(1, 2) = 3)\).

c. \(\mathbb{E}(T_2 \mid N(2) = 1)\).
2. Customers in a Store

Consider two independent Poisson processes with rates $\lambda_1$ and $\lambda_2$, which measure the number of customers arriving in store 1 and 2.

a. What is the probability that a customer arrives in store 1 before any arrives in store 2?

b. What is the probability that in the first hour, a total of exactly 6 customers arrive in the two stores?

c. Given that exactly 6 have arrived in total at the two stores, what is the probability that exactly 4 went to store 1?
3. Spatial Poisson Process

A two-dimensional Poisson process of rate $\lambda > 0$ is a process of randomly occurring *special points* in the plane such that

i. For any region of area $A$, the number of special points in that region is distributed as a Poisson with mean $\lambda A$, and

ii. The number of special points in non-overlapping regions is independent.

For such a process, consider a fixed arbitrary location in the plane, and let $X$ denote its distance from its nearest special point, where the distance between two points is the usual

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$ 

a. Show that $\mathbb{P}(X > t) = \exp(-\lambda \pi t^2)$ for $t > 0$.

b. Show that $\mathbb{E}(X) = \frac{1}{2\sqrt{\lambda}}$. 