1. Syllabus

Welcome to EECS126! Please read the “Course Information” page in eeCS126.org, and answer these questions.

(a) What percentage of your grade is midterm 1? What about midterm 2, and the final?
(b) Can your final grade replace both your midterm 1 and midterm 2 grade?
(c) Which day of the week and what time are homeworks due?
(d) Which day of the week and what time are labs due?
(e) If you turn in a late self-grade, will it be accepted? If you don’t turn in a self-grade on time, what will your corresponding homework/lab score be?
(f) What are the dates for MT1, MT2, and the final?
(g) How many lab and homework drops do you have?
(h) What is the minimum score you need to get for full credit on each homework?
2. Balls & Bins

Let $n \in \mathbb{Z}_{>1}$. You throw $n$ balls, one after the other, into $n$ bins, so that each ball lands in one of the bins uniformly at random. What is an appropriate sample space to model this scenario? What is the probability that exactly one bin is empty?
3. Coin Flipping & Symmetry

Alice and Bob have $2n + 1$ fair coins (where $n \geq 1$), each coin with probability of heads equal to $1/2$. Bob tosses $n + 1$ coins, while Alice tosses the remaining $n$ coins. Assuming independent coin tosses, show that the probability that, after all coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

*Hint:* Consider the event $A = \{\text{more heads in the first } n + 1 \text{ tosses than the last } n \text{ tosses}\}$. 
4. Superhero Basketball

Superman and Captain America are playing a game of basketball. At the end of the game, Captain America scored \( n \) points and Superman scored \( m \) points, where \( n > m \) are positive integers. Supposing that each basket counts for exactly one point, what is the probability that after the start of the game (when they are initially tied), Captain America was always strictly ahead of Superman? (Assume that all sequences of baskets which result in the final score of \( n \) baskets for Captain America and \( m \) baskets for Superman are equally likely.)

*Hint:* Think about symmetry. First, try to figure out which is more likely: Superman scored the first point and there was a tie at some point in the game or Captain America scored the first point and there was a tie at some point in the game?
5. Expanding the NBA

The NBA is looking to expand to another city. In order to decide which city will receive a new team, the commissioner interviews potential owners from each of the $N$ potential cities ($N$ is a positive integer), one at a time. Unfortunately, the owners would like to know immediately after the interview whether their city will receive the team or not. The commissioner decides to use the following strategy: she will interview the first $m$ owners and reject all of them ($m \in \{1, \ldots, N\}$). After the $m$th owner is interviewed, she will pick the first city that is better than all previous cities. The cities are interviewed in a uniformly random order. What is the probability that the best city is selected? Assume that the commissioner has an objective method of scoring each city and that each city receives a unique score.

You should arrive at an exact answer for the probability in terms of a summation. Approximate your answer using $\sum_{i=1}^{n} i^{-1} \approx \ln n$ and find the optimal value of $m$ that maximizes the probability that the best city is selected. You can also say $\ln(n - 1) \approx \ln n$.

*Hint:* Consider the events $B_i = \{i$-th city is the best$\}$ and $A = \{\text{best city is chosen}\}$.
6. Passengers on a Plane

There are $N$ passengers in a plane with $N$ assigned seats ($N$ is a positive integer), but after boarding, the passengers take the seats randomly. Assuming all seating arrangements are equally likely, what is the probability that no passenger is in their assigned seat? Compute the probability when $N \to \infty$.

*Hint:* Use the inclusion-exclusion principle and the power series $e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!}$. 
7. Upperclassmen

You meet two students in the library. Assume each student is an upperclassman and under-
classman with equal probability and each student takes 126 with probability $\frac{1}{10}$, independent of
each other and independent of their class standing. What is the probability that both students
are upperclassmen given that at least one of them is an upperclassmen who is currently taking
EECS126?