Midterm 1

Last Name	First Name	SID

Left Neighbor First and Last Name	**Right Neighbor** First and Last Name

***Rules*.**

- **Write in your SID on every page to receive 1 point.**
- **Unless otherwise stated, all your answers need to be justified and your work must be shown. Answers without sufficient justification will get no credit.**
- You have 80 minutes to complete the exam. (DSP students with $X\%$ time accommodation should spend $80 \cdot X\%$ time on completing the exam).
- This exam is closed-book. You may reference one double-sided sheet of notes. No calculator or phones allowed.
- Remember the Berkeley Honor Code: "*As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*" Any violation of academic integrity will be taken seriously, and could result in disciplinary consequences.

Problem	out of
SID	1
Problem 1	10
Problem 2	15
Problem 3	15
Problem 4	20
Problem 5	20
Problem 6	24
Problem 7	15
Total	120

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1 Something Good [7 + 3 points]

a) What's something you are looking forward to this semester? (doesn't have to be related to this class)

b) Write down as many passcodes from lecture as you can remember.

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2 Monkey Business [5 + 10 points]

A monkey types on a keyboard with 27 keys (corresponding to letters a-z, plus a period '.').

a) Assuming the monkey types each character independently and uniformly at random, what is the probability they type "class." on their first try? (Note the period at the end of the word 'class')

b) Let $\ell(i)$ denote the number of letters in the ith word in the English dictionary (all words are composed of only letters a-z, no periods). For example, if 'aardvark' is the 3rd word in the dictionary, then $\ell(3) = 8$. Assuming there are N words in the dictionary, use the axioms of probability to show that

$$\sum_{i=1}^{N} \left(\frac{1}{27}\right)^{\ell(i)} \leq 27.$$

Hint: Define N disjoint events.

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3 Bear Territory [15 points]

A zoologist observes $B \sim \text{Poisson}(\mu)$ bears living on the prairie. Bear i has a territory with "range parameter" $R_i \sim \mathcal{N}(r, \sigma^2)$. Conditioned on R_i, the territory of bear i has area $X_i \sim \text{Uniform}(R_i^2, 3R_i^2)$, independent of the number of bears B. None of the bear territories overlap. What is the expected total territory area T occupied by all bears on the prairie?

4 Noisy Observations [10 + 10 points]

Let X and Z be independent standard normals. Suppose we consider

$$Y = \alpha X + Z,$$

where $\alpha \in \mathbb{R}$.

a) Find the conditional density $f_{X|Y}(x|y)$. What is the distribution of X given $\{Y = y\}$? Specify any parameters of the distribution as needed.

b) Compute the conditional expectations $\mathrm{E}[X|Y = y]$ and $\mathrm{E}[Y|X = x]$.

Hint: The density of $\mathcal{N}(\mu, \sigma^2)$ is $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

5 Lucky Draws [5 + 5 + 10 points]

There are n red balls and n green balls in a bucket. You sequentially draw all $2n$ balls, without replacement from the bucket, and record the sequence of the colors you observe. A draw is considered a "lucky draw" if it is the same color as the previous draw. Let N denote the number of lucky draws after drawing all balls.

a) Formulate a suitable probability space for this problem.

b) Express the random variable N as an explicit function $N(\omega)$ of the underlying sample $\omega \in \Omega$. It is acceptable to leave your answer in the form of a sum.

c) Find $\mathrm{E}[N]$.

6 Nth Times a Charm [8 + 8 + 8 points]

You have a coin that turns heads with probability p and tails with probability $1 - p$. For a given integer $k \geq 1$, let N denote the number of independent flips until you see exactly k heads.

a) Find the pmf of N.

b) Compute the moment generating function $M_N(t)$.

c) Compute the variance of N.

Note: You might find the series $\sum_{n\geq 1} n^2 q^n = \frac{q(1+q)}{(1-q)^3}$ (for $|q| < 1$) helpful in part (c).

7 Reasonable Grading [15 points]

A professor claims that they can grade exam problems simply by sniffing the page on which the solution is written. Each problem is graded on a scale from 0-4 points (4 = A, 3 = B, 2 = C, 1 = D, 0 = F). When the professor smells 'A' work, they immediately recognize it as 'A' work; likewise, when the professor smells 'F' work, they also recognize it without mistake. However, when the student solution deserves 1,2 or 3 points, the smells can be hard to discern, and the professor adds 1 point, subtracts 1 point, or gets the score correct, each with probability 1/3, independent of all other problems.

Throughout the semester, 25 exam problems are graded for each student, and the scores are averaged and then rounded to the nearest integer determine the course letter grade (4 = A, 3 = B, etc.).

Suppose that a particular student deserved an average score of 3.1 (i.e., this would have been their score if the professor made no mistakes in grading the questions). Find a reasonably good numerical upper bound on the probability that a student receives a lower grade than the B they deserve.

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