Problem 1. Maximum Likelihood Estimation.
In problem 4 in homework 4 (the polling problem), we argued that for k samples $Y_i, i = 1, 2, \ldots, k$, a reasonable estimate of a fixed parameter q, the fraction of people that will vote for Kerry, is $\hat{q} = \frac{1}{k}\sum_{i=1}^{k} Y_i$. We used the results from problem 6 in homework 3 to support our claim. Now we want to make a close look to see why our original estimate is reasonable? Theoretically, why it makes sense? Is there any criteria it optimizes?.
a) In the problem, q is some fixed number to be estimated, we collect a bunch of samples whose distribution are functions of q. Based on those samples, we try to produce an estimate of p. How to model the problem?
b) Given the observations $Y_i, i = 1, 2, \ldots, k$, what is the ML estimation of q, denoted as $\hat{q}$?

Problem 2.
In class, we saw how we can estimate a continuous random variable X given one observation Y; we will see how to do the MMSE estimation for X given two observations Y_1 and Y_2. The signal X follows $N(0, \sigma_x^2)$. Two observations are:

$$\begin{aligned} Y_1 &= X + Z_1, \\ Y_2 &= X + Z_2, \end{aligned}$$

where Z_1 and Z_2 are i.i.d. Gaussian noise with mean zero and variance σ^2.
a) Are there sufficient information for you to come up with the MMSE estimate of X, denoted as $\hat{X}$? If yes, go to part b), if no, give additional assumptions and then go to part b).
b) What is the fundamental of MMSE estimation, i.e. essentially what you want to compute to give out the MMSE estimation?
c) For this particular problem, compute the MMSE estimate $\hat{X}$, and the estimation error.
d) Generalize your result to n observations case. What happens to estimation error when n is large? Given an intuitive explanation.

Problem 3.
a) Suppose now X and Y are i.i.d. with unknown distribution, and we observe $Z = X + Y$. What is the MMSE estimate of X?
b) An amplifier amplifies input X by a factor of μ, thus outputting $Y = \mu X$. However, due to defeat in the device, the amplifying factor is not a fixed number, but a r.v. Z following exponential distribution with mean μ. Suppose now X follows a uniform distribution in $[a, b]$, what is the MMSE estimate $\hat{X}$? What is the estimation error?