EE 126: Probability and Random Processes

Problem Set 7

Due: March 18 in class

Marks for each questions are specified.

[15] 1. Chapter 3, Problem 21 in http://www.athenasc.com/CH3-prob-supp.pdf.

[15] 2. Chapter 4, Problem 10 in http://www.athenasc.com/CH4-prob-supp.pdf.

[15] 3. (a) In class, we stated that

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(x, z-x)dx$$

where $Z = X + Y$. Justify this statement in terms of probabilities of events. (Hint: you may want to draw a picture.)

(b) Let X be a discrete rv with a pmf p_X and Y be a continuous rv with a pdf f_Y. X and Y are independent rv's. Is $X + Y$ discrete, continuous, or mixed? Find the pmf or pdf of $X + Y$ in terms of p_X and f_Y.

[15] 4. Consider the problem we discussed in class where there are two cards, one marked with a payoff X and one with a payoff $2X$, where X is a random variable uniformly distributed in $[0, 1000]$. X is unknown to you *a priori*. You randomly pick a card and see the payoff. You have the option of keeping the payoff or switching to the other card. Determine the optimal strategy.

[30] 5. a) Suppose you have access to a rv $X \sim U[0, 1]$. In class it was shown how to use X to generate a continuous rv with a given distribution. Adapt the method to generate a discrete finite-valued rv with a given pmf. (You may want to start with a Bernouilli rv with probability p to be 1 and probability $1 - p$ to be 0.)

b) In MATLAB, a uniform rv can be generated by calling the routine "rand(1)". Generate a binomial rv with parameters $n = 10$ and $p = 0.2$ in two ways:

i) By directly using the method you gave in (a).

ii) By first using the method in (a) to generate a Bernouilli rv, and then generate the Binomial rv as a sum of Bernouilli rv's.

How do you test whether the rv you generated has the desired pmf? Hand in both the MATLAB code and numerical evidence to support the fact that you've got the right rv.