UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Discussion Section 7
Spring 2006

Problem 7.1

[Midterm 2, Fall 04] Widgets are packed into cartons which are packed into crates. The weight (in pounds) of a widget is a continuous random variable with pdf

$$f_X(x) = \lambda e^{-\lambda x} \text{ for x} > 0$$

The number of widgets in any carton, K, is a random variable with the pmf

$$p_K(k) = e^{-\mu}\frac{\mu^k}{k!} \text{ for } k = 0, 1, 2, \ldots$$

The number of cartons in a crate, N, is a random variable with pmf

$$p_N(n) = p^{n-1}(1-p) \text{ for n = 1,2,3,}\ldots$$

The random variables X, K and N are mutually independent. Determine:

(a) The probability that a randomly selected crate contains exactly one widget

(b) The conditional pdf for the total weight of widgets in a carton given that the carton contains less than two widgets

(c) The transform of the pdf for the total weight of the widgets in a crate

Problem 7.2

Iwana Passe is taking a quiz with 12 questions. The amount of time she spends answering question i is T_i, and is exponentially distributed with $E[T_i] = \frac{1}{3}$ hour. The amount of time she spends on any particular question is independent of the amount of time she spends on any other question. Once she finishes answering a question, she immediately begins answering the next question.

Let N be the total number of questions she answers **correctly**.

Let X be the total amount of time she spends on questions that she answers **correctly**.

For parts (a) and (b), suppose we know she has probability $\frac{2}{3}$ of getting any particular quiz question correct, independently of her performance on any other quiz question.

(a) Find the expectation X.

(b) Assuming we know she spent at least $\frac{1}{6}$ of an hour on each question, find the transform of X.

For parts (c) and (d), suppose she has a fixed probability P of getting any particular quiz question correct, independently of her performance on any other quiz question, and with P uniformly distributed between 0 and 1. Assume P is the same value for all questions.

(c) Find the expectation N.

(d) Assuming there is only one question on the quiz, find the transform of N.

Problem 7.3

Harry and Larry approach each other from very far away. Harry will see Larry at a distance that is exponentially distributed with parameter 1 km, while Larry will see Harry at a distance that is uniformly distributed between 0 and 1 km. Use convolution to find the PDF of X, the distance during which only one of the two persons will be seeing the other.

Problem 7.4

Suppose X is a geometric random variable with parameter p where p is uniformly distributed from 0 to $\frac{n-1}{n}$. Define a new random variable Z by $Z = E[X|P = p]$. Find and interpret $E[Z]$, and then compute $\lim_{n\to\infty} E[Z]$.