UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 1
Spring 2006

Issued: Thursday, January 17, 2006 **Due:** Thursday, January 26, 2006

Reading: Berstsekas & Tsitsiklis, §1.1, §1.2. and §1.3

Problem 1.1
For three tosses of a fair coin, define the sample space. Find the probabilities of the following events:

(a) the sequence HHH?

(b) the sequence HTH?

(c) seeing two heads and one tail?

(d) the outcome "More heads than tails"?

Problem 1.2
Given two events A and B on a common sample space, give expressions for the following probabilities in terms of $\mathbb{P}(A)$, $\mathbb{P}(B)$ and $\mathbb{P}(A \cap B)$.

(a) The probability that *at least one* of A or B occurs?

(b) The probability that *exactly one* of A and B occurs?

Which one of these probabilities is equal to $\mathbb{P}\Big((A \cap B^c) \cup (A^c \cap B)\Big)$?

Problem 1.3
A four-sided die is rolled repeatedly, until the first time (if ever) that an even number is obtained. What is the sample space for this experiment?

Problem 1.4
Suppose that computer programs can have two types of bugs: bug A and bug B. Suppose that when monkeys with laptops type in programs, 40% of programs have bug A, 70% of programs have bug B, and 90% of programs have bug A *or* bug B. Suppose a poorly written computer program causes a computer to crash if and only if the program has bug A and *not* bug B. If I randomly choose a program generated by monkeys and run it, what is the probability that my computer will crash? Illustrate your reasoning with a Venn diagram.

Problem 1.5
A backyard astronomer uses his telescope to observe shooting stars. Consider the sample space $\{0, 1, 2, 3, \ldots\}$, corresponding to the number of shooting stars that he sees on any given night.

(a) Suppose that he tells you that the probability of seeing n stars is $\mathbb{P}(\text{see } n \text{ stars}) = \frac{1}{n+10}$ for $n = 0, 1, 2, \ldots$. Does this define a valid probability law? Why or why not?

(b) What if he changes the rule to $\mathbb{P}(\text{see } n \text{ stars}) = \frac{1}{2^{n+1}}$? Why or why not?

Problem 1.6
Alice and Bob play the following game. Alice starts with a candy cane with unit length, and randomly breaks it, and keeps *one* piece. Bob starts with a different candy cane of unit length, randomly breaks it, and keeps *both* pieces. Assuming that all possible pairs of breaking points for the two candy canes are equally likely, what is the probability that Alice's one piece is longer than the product of the lengths of Bob's two pieces?

Problem 1.7
A six-sided die is loaded in a way that each even face is twice as likely as each odd face. Construct a probabilistic model for a single roll of this die, and find the probability that a 1, 2, or 3 will come up.

Problem 1.8
Consider the sets $A_n = \{x \in \mathbb{R} \mid 0 \leq x \leq 1 - \frac{1}{n+1}\}$ for $n = 1, 2, 3, \ldots$. What is $\cap_{n=1}^{\infty} A_n$? What is $\cup_{n=1}^{\infty} A_n$? Justify your answers.