UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 8

Spring 2006

Issued: Friday, March 17, 2006 **Due:** Friday, March 24, 2006

Reading: Bertsekas & Tsitsiklis, §4.1–4.5, §4.7

Problem 8.1

Consider two independent and identically distributed random variables X_1 and X_2. Compute the PDF (or PMF) of $Y = X_1 + X_2$, assuming that

(a) X_1 and X_2 have an exponential density with parameter λ.

(b) X_1 and X_2 have a geometric PMF with parameter p.

Problem 8.2

(Optional; not to be graded) Suppose now that $Y = X_1 + \ldots + X_k$, where $X_1, X_2, \ldots, X_k$ are k independent and identically distributed random variables.

(a) If the $X_1, X_2, \ldots, X_k$ have an exponential density with parameter λ, show that the pdf of Y is

$$f_Y(y) = \frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!} \quad y \geq 0$$

which is known as the Erlang pdf of order k.

(b) If the $X_1, X_2, \ldots, X_k$ have an geometric distribution with parameter p, show that the PMF of Y is

$$P(Y = n) = \binom{n-1}{k-1} p^k (1-p)^{n-k} \quad n = k, k+1, \ldots$$

which is known as the Pascal PMF of order k.

Problem 8.3

Consider two independent random variables X and Y. Let $f_X(x) = 1 - x/2$ for $x \in [0, 2]$ and 0 otherwise. Let $f_Y(y) = 2 - 2y$ for $y \in [0, 1]$ and 0 otherwise. Compute the PDF of $W = X + Y$.

Problem 8.4

You enter the first floor of a castle with 78 floors, with the goal of reaching (and stealing) a huge treasure pile on the top floor. However, guards patrol the castle, and on each floor of the castle, there is a probability 1/2 of bumping into a guard. Bumping into a guard results in a change in your energy (the person you bump into may be friendly and give you something to eat, or may be nasty and you may have to put up a fight, etc). Your change in energy in each such interaction is distributed as a normal random variable with mean 1 unit and standard deviation 1/2 units; it is independent of how many guards you bump into and of your change in energy in other interactions. No matter how your energy changes on any given floor, you always proceed to the next floor. Let X be your total change in energy by the time you reach the treasure.

(a) Use the total probability law to find the PDF and transform associated with X. Is X normal?

(b) Find the transform associated with X by viewing X as a sum of a random number of random variables.

Problem 8.5

N male-female couples at a dinner party play the following game. Each member of the couple, writes his/her name on a piece of paper (so there are a total of $2N$ pieces of paper). Then men throw their paper into hat A, while the women throw their paper in hat B. Once all the papers are in, the host draws a piece of paper from each hat, and the two people chosen are dance partners for the rest of the night. The host continues in this fashion until all $2N$ poeple are paired up for night. Let M be the number of couples that are reunited by this game. Find $E[M]$ and $Var(M)$.

Problem 8.6

The time at which a lazy Berkeley professor (who will remain nameless) arrives on campus each day is uniformly distributed between noon and 2pm. The professor's one graduate student arrives on campus promptly at 8am. Each day the student goes to the professor's office to meet at a time that is uniformly distributed between noon and 5pm. If the professor is not in his office when the student arrives, the student goes home and drinks beer for the rest of the day. If the professor is in, they meet for an amount of time that is uniformly distributed between the time the student arrives and 5pm and then they both go home. If the lazy professor arrives and finds that his student has gone home, he immediately goes home to sleep for the rest of the day.

(a) What is the expected value of the number of hours that the student is on campus each day?

(b) What is the expected value of the number of hours that the professor is on campus each day?

Problem 8.7

Random variables X_1, X_2 describe the coordinates of a point on the plane. Suppose that X_1, X_2 have bivariate normal joint distribution, with $\mu_{x_1} = \mu_{x_2} = 0$ and $\sigma^2_{x_1} = \sigma^2_{x_2} = 1$ and $\rho = 0$. Find the probability that this point lies within α of the origin.