UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 9

Spring 2006

Issued: Friday, April 14, 2006 **Due:** Friday, April 21, 2006

Reading: Bertsekas & Tsitsiklis, Chapter 5, §6.1

Problem 9.1

Fred is giving out samples of dog food. He makes calls door to door, but he leaves a sample (one can) only on those calls for which the door is answered *and* a dog is in residence. On any call the probability of the door being answered is 3/4, and the probability that any household has a dog is 2/3. Assume that the events "Door answered" and "A dog lives here" are independent and also that the outcomes of all calls are independent.

(a) Determine the probability that Fred gives away his first sample on his third call.

(b) Given that he has given away exactly four samples on his first eight calls, determine the conditional probability that Fred will give away his fifth sample on his eleventh call.

(c) Determine the probability that he gives away his second sample on his fifth call.

(d) Given that he did not give away his second sample on his second call, determine the conditional probability that he will leave his second sample on his fifth call.

(e) We will say that Fred "needs a new supply" immediately *after* the call on which he gives away his last can. If he starts out with two cans, determine the probability that he completes at least five calls before he needs a new supply.

(f) If he starts out with exactly m cans, determine the expected value and variance of D_m, the number of homes with dogs which he passes up (because of no answer) before he needs a new supply.

Problem 9.2

All ships travel at the same speed through a wide canal. Eastbound ship arrivals at the canal are a Poisson process with an average arrival rate λ_E ships per day. Westbound ships arrive as an independent Poisson process with average arrival rate λ_W per day. An indicator at a point in the canal is always pointing in the direction of travel of the most recent ship to pass it. Each ship takes t days to traverse the length of the canal.

(a) Given that the pointer is pointing west:

  (i) What is the probability that the next ship to pass it will be westbound?

  (ii) What is the PDF for the remaining time until the pointer changes direction?

(b) What is the probability that an eastbound ship will see no westbound ships during its eastward journey through the canal?

(c) We begin observing at an arbitrary time. Let V be the time we have to continue observing until we see the seventh eastbound ship. Determine the PDF for V.

Problem 9.3

On each trial of a game, both Don and Greg simultaneously, but independently, flip biased coins. On each trial, the probability that Don's flip results in a head is p_D, while the probability that Greg's flip results in a head is p_G.

(a) Given that the flips on a particular trial resulted in 2 heads, find the PMF for M, the number of additional trials up to and including the next trial on which 2 heads result.

(b) Given that the flips on a particular trial resulted in *at least* one head, find the probability that Don flipped a head on the trial.

(c) Starting from a trial on which no heads result, find the probability that Don's next flip of a head will occur *before* Greg's next flip of a head.

Problem 9.4

Consider a chess tournament between Al and Bo. Let x_i denote the duration of the ith game and assume that the x_i's are IID exponentially distributed random variables with PDF $f_x(x) = \lambda e^{-\lambda x}$ $x \geq 0$. Suppose that each game (independently of all other games, and independently of the length of the games) is won by Al with probability p, by Bo with probability q, and is a draw with probability $1 - p - q$. The first player to win N games is defined to be the winner. It may be helpful to consider the match up to the point of winning as being embedded in an unending sequence of games.

(a) Find the PDF for t, the time from the beginning of the match to the time of completion of the first game that is won (i.e., that is not a draw). Characterize the process of the total number of games won, the process of the number of games won by Al, and the number won by Bo.

For the remainder of the problem, assume that the probability of a draw is zero; i.e., that $p + q = 1$.

(b) If we consider the unending sequence of games, how many of the first $2N - 1$ games must be won by Al in order to win the match?

(c) What is the probability that Al wins the match? Your answer can be left in the form of a summation but should not involve any integrals. (Hint: consider the unending sequence of games and use part b.)

(d) Let v be the time at which the match is completed (i.e., either Al or Bo wins). Find the cumulative distribution function of v. Again, your answer can be left in the form of a summation.