

Discussion 10

Spring 2016

Date: Wednesday, April 13, 2016

Problem 1. (Fall 2008, MT2) Given $X \in \{0, 1\}$, the random variable Y is exponentially distributed with rate $3X + 1$.

- (a) Assume $P(X = 1) = p$ and $P(X = 0) = 1 - p$. Find the MAP estimate of X given Y .
- (b) Find the MLE of X given Y .
- (c) Solve the hypothesis testing problem of X given Y with a probability of false alarm at most 0.1. That is, find \hat{X} as a function of Y that maximizes $P[\hat{X} = 1|X = 1]$ subject to $P[\hat{X} = 1|X = 0] \leq 0.1$.
- (d) For what value of p does one have the same solution for (a) and (c)?

Problem 2. Let $Y \in \{A, B, C\}$ such that $P(Y = A|X = 1) = 0.2, P(Y = B|X = 1) = 0.2, P(Y = C|X = 1) = 0.6$ and $P(Y = A|X = 0) = 0.2, P(Y = B|X = 0) = 0.5, P(Y = C|X = 0) = 0.3$. We would like to solve a hypothesis testing with probability of false alarm at most β .

- (a) Find the likelihood ratio.
- (b) Find PCD and PFA for $\lambda = 1, 1.4, 2, 2.1$.
- (c) Draw the ROC