UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

**Problem Set 9**
Spring 2016

**Issued:** Thursday, March 31, 2016 **Due:** 11:59PM, Thursday, April 7, 2016

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*Problem* 1. Midterm 2.

*Problem* 2. The random variable $X$ is exponentially distributed with mean 1. Given $X$, the random variable $Y$ is exponentially distributed with rate $X$ (with mean $1/X$).

(a) Find $MLE[X|Y]$;

(b) Find $MAP[X|Y]$.

*Problem* 3. It will be useful to work on this problem in conjunction with Q3 of Lab 9. The stochastic block model (SBM), as defined in Lab 9 is a random graph $G(n, p, q)$ consisting of two communities of size $\frac{n}{2}$ each such that the probability an edge exists between two nodes of the same community is $p$ and the probability an edge exists between two nodes in different communities is $q$, where $p > q$. The goal of the problem is to exactly determine the two communities given only the graph. Show that the MAP-decision rule is equivalent to finding the min-bisection of the graph (ie the split of $G$ into two groups of size $\frac{n}{2}$ that has the minimum edge weight across the partition).

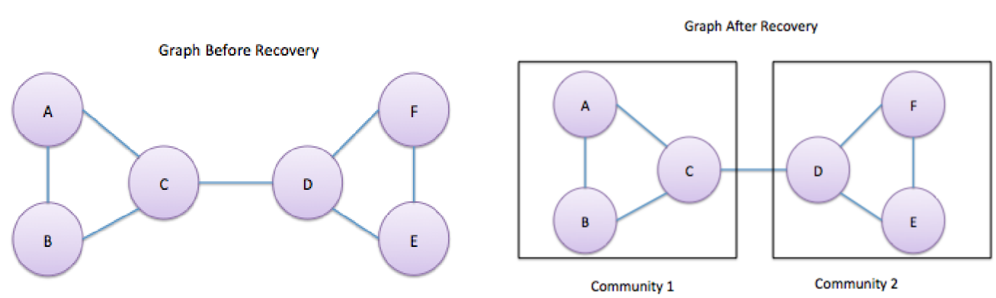


Figure 1: An example of a graph before and after recovery

*Problem* 4. In this problem, we use similar settings which were considered in HW02. Consider a random bipartite graph, $G_1$, with $K$ left nodes and $M$ right nodes. Each of the $KM$ possible edges of this graph is connected with probability $p$ independently. In the following problems, we consider the situations when $M$ and $K$ are large and $Mp$ and $Kp$ are constants.
*Hint:* Use the Poisson distribution to approximate binomial distribution and apply law of large numbers.

(a) A singleton is a right node of degree one. As $M$ and $K$ get large, how many left nodes are connected to right nodes which are singletons?

(b) A doubleton is a right node of degree two. As $M$ and $K$ get large, how many doubletons do we have?

(c) We call 2 doubletons distinct, if they are not connected to the same 2 left nodes. As $K$ and $M$ get large, what is the probability that two doubletons are distinct?

*Problem* 5. Consider the same setting as the previous problem.

(a) Let $M_s$ be the number of doubletons for which both of the left nodes are also connected to singletons. Find $M_s$ as $K$ and $M$ get large.

(b) We construct another random graph, $G_2$, as follows. Let $K_s$ be the number of left nodes that are connected to singletons, which you calculated in part (a). Graph $G_2$ has $K_s$ nodes corresponding to these left nodes. Two nodes in $G_2$ are connected if there is a doubleton in $G_1$ that is connected to those left nodes. Thus, $G_2$ has $M_s$ edges which you calculated in part (d). Argue that $G_2$ is equivalent to an Erdos-Renyi random graph.

(c) An Erdos-Renyi random graph $G(N, q)$ has a giant component of size linear in $N$ if $Nq > 1$. A giant component is the largest set of nodes in the graph that is connected. Suppose that $M = 4K$. Find a condition on $p$ as a function of $K$ such that $G_2$ has a giant component.