UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 10
Spring 2016

Issued: Thursday, April 7, 2016 **Due:** 11:59pm Thursday, April 14, 2016

Problem 1. Consider the following communication channel. There is an i.i.d. source that generates symbols $\{1, 2, 3, 4\}$ according to a prior distribution $\pi = [p_1, p_2, p_3, p_4]$. The symbols are modulated by QPSK scheme, i.e. they are mapped to constellation points $(\pm 1, \pm 1)$. The communication is on a baseband Gaussian channel, i.e. if the sent signal is $(x_1, x_2),\ x_i \in \{-1, 1\}$, the received signal is

$$y_1 = x_1 + z_1,$$

$$y_2 = x_2 + z_2,$$

where z_1 and z_2 are independent $N(0, \sigma^2)$ random variables. We let the symbols $\{1, 2, 3, 4\}$ correspond to $(1, 1)$, $(-1, 1)$, $(-1, -1)$, and $(-1, 1)$, respectively. Find the MAP detector of this communication channel.

Problem 2. Customers arrive to a store according to a Poisson process of rate 1. The store manager learns of a rumor that one of the employees is sending $\frac{1}{2}$ of the customers to the rival store. Refer to hypothesis $X = 1$ as the rumor being true, that one of the employees is sending every other customer arrival to the rival store and hypothesis $X = 0$ as the rumor being false, where each hypothesis is equally likely. Assume that at time 0, there is a successful sale. After that, the manager observes $S_1, S_2, \ldots, S_n$ where S_i is the time of the ith subsequent sale. Derive the MAP rule to determine whether the rumor was true or not.

Problem 3. You are testing a digital link that corresponds to a BSC with some error probability $\epsilon \in [0, 0.5)$.

(a) Assume you observe the input and the output of the link. How do you find the MLE of ϵ?

(b) You are told that the inputs are i.i.d. bits that are equal to 1 with probability 0.6 and to 0 with probability 0.4. You observe n outputs. How do you calculate the MLE of ϵ.

(c) The situation is as in the previous case, but you are told that ϵ has pdf $4 - 8x$ on $[0, 0.5)$. How do you calculate the MAP of ϵ given n outputs.

Problem 4. You are trying to detect whether voltage V_1 or voltage V_2 was sent over a channel with independent Gaussian noise $Z \sim N(V_3, \sigma^2)$. Assume that both voltages are equally likely to be sent.

(a) Derive the MAP detector for this channel.

(b) Using the Gaussian Q-function, determine the average error probability for the MAP detector.

(c) Suppose that the average transmit energy is $\frac{V_1^2+V_2^2}{2}$ and that the average transmit energy is constrained such that it cannot be more than E. What voltage levels V_1, V_2 should you choose to meet this energy constraint but still minimize the average error probability?

Problem 5. Consider a hypothesis testing problem that if $X = 0$, you observe a sample of $N(\mu_0, \sigma^2)$, and if $X = 1$, you observe a sample of $N(\mu_1, \sigma^2)$. Find the Neyman-Pearson test for false alarm α, i.e. $\Pr(\hat{X} = 1 | X = 0) \leq \alpha$.