UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 1
Spring 2017

Issued: Tuesday, January 17, 2017 **Due:** 8am, Thursday, January 26, 2017

Problem 1. Consider a fair roll of a 4-sided dice. Find three events which are pairwise independent but not mutually independent.

Problem 2. A set of 50 items is inspected by testing 4 randomly selected items (without replacement); if all 4 are not defective, the set is accepted. Suppose that 5 items in the set are defective, what is the probability that the set is accepted?

Problem 3. Consider the following scenario: two envelopes are placed in front of you, each of which contains a different positive, integer amount. You randomly select one of the two envelopes and peek inside so that you see the amount in that envelope. You may either keep the amount in this envelope, or switch to the other envelope, but at that point your choice is fixed. Your friend tells you that if you toss a coin until you see a heads and add $\frac{1}{2}$ to the number tosses it took to see a heads *and* that number is greater than the number in the envelope, you should switch and select the other envelope. Is your friend correct?

Problem 4. (a.) Suppose that there are two coins in front of you such that one is fair, and the other is biased such that the probability of heads is $\frac{3}{4}$. You pick one of the two at random and flip it 4 times. The coin comes up heads twice. What is the probability that the coin is fair?

(b.) Now suppose that there is only one coin in front of you and it has unknown bias. Can you devise a scheme to use the biased coin to make a fair decision?

Problem 5. There are m people passing a ball to each other. At the beginning, Bob has the ball. In each round, the person who has the ball passes the ball equally likely to one of the other $m - 1$ people. What is the probability that in the nth round, the ball is passed from Bob?

Problem 6. Consider a sphere that has $\frac{1}{10}$ of its surface is colored blue, and the rest is red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

Problem 7. The Donald and Hillary are playing a game of basketball. At the end of the game, Hillary scored n points and The Donald scored m where $n > m$ (Hillary defeated The Donald). Supposing that each basket counts for exactly 1 point, what is the probability that during the course of the game, Hillary was always *strictly* ahead of The Donald?

Problem 8. The NBA is looking to expand to another city. In order to decide which city will receive a new team, the commissioner interviews potential owners from each of the N potential cities one at a time. Unfortunately, the owners would like to know immediately after the interview whether their city will receive the team or not. The commissioner decides to use the following strategy: she will interview the first m owners and reject all of them. After the mth owner is interviewed, she will pick the first city that is better than all previous cities. What is the probability that the best city is selected? Assume that the commissioner has an objective method of scoring each city and that each city receives a unique score.