UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 9
Spring 2018

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1. **Continuous-Time Markov Chains: Introduction**

Consider the continuous-time Markov process with state space $\{1, 2, 3, 4\}$ and the rate matrix

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

(a) Find the stationary distribution p of the Markov process.

(b) Find the stationary distribtion π of the jump chain, i.e., the discrete-time Markov chain which only keeps track of the jumps of the CTMC. Formally, if the CTMC $(X(t))_{t\geq 0}$ jumps at times $T_1, T_2, T_3, \ldots$, then the DTMC is defined as $(Y_n)_{n=1}^{\infty}$ where $Y_n := X_{T_n}$.

(c) Suppose the chain starts in state 1. What is the expected amount of time until it changes state for the first time?

(d) Again assume the chain starts in state 1. What is the expected amount of time until the chain is in state 4?

2. $M/M/2$ **Queue**

A queue has Poisson arrivals with rate λ. It has two servers that work in parallel. When there are at least two customers in the queue, two are being served. When there is only one customer, only one server is active. The service times are i.i.d. exponential random variables with rate μ. Let $X(t)$ be the number of customers either in the queue or in service at time t.

(a) Argue that the process $(X(t), t \geq 0)$ is a Markov process.

(b) Draw the state transition diagram.

(c) Find the range of values of μ for which the Markov chain is positive-recurrent and for this range of values calculate the stationary distribution of the Markov chain.

3. **Particles Moving on a Checkerboard**

There are 1278 particles on a 100×100 checkerboard. Each location on the checkerboard can have at most one particle. Each particle, at rate 1, independently over the particles, attempts to jump, and when it does it tries to move in one of the four directions, up, down, left, and right, equiprobably. However, if this movement would either take it out of the checkerboard or

onto a location that is already occupied by another particle then the jump is suppressed, and nothing happens.

What is the stationary distribution of the configuration of the particles on the checkerboard?