UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Problem Set 8
Spring 2018

Issued: Friday, March 9, 2018 **Due:** Wednesday, March 14, 2018

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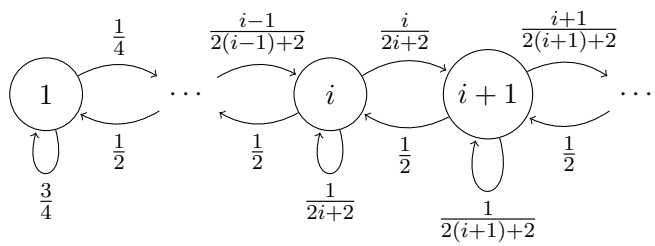
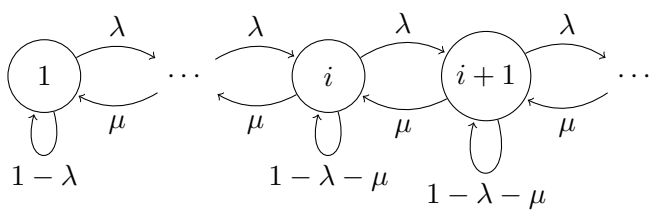
1. **Markov Chains with Countably Infinite State Space**

   (a) Consider a Markov chain with state space $\mathbb{Z}_{>0}$ and transition probability graph:



   Show that this Markov chain has no stationary distribution.

   (b) Consider a Markov chain with state space $\mathbb{Z}_{>0}$ and transition probability graph:



   Assume that $0 < \lambda < \mu$ and $0 < \lambda + \mu \leq 1$. Show that the probability distribution given by

$$\pi(i) = \left(\frac{\lambda}{\mu}\right)^{i-1}\left(1 - \frac{\lambda}{\mu}\right), \text{ for } i \in \mathbb{Z}_{>0},$$

   is a stationary distribution of this Markov chain.

2. **Choosing Two Good Movies**

   You have a database of a countably infinite number of movies. Each movie has a rating that is uniformly distributed in $\{0, 1, 2, 3, 4, 5\}$ and you want to find two movies such that the sum of their rating is greater than 7.5. Assume that you choose movies from the database one by one and keep the movie with the highest rating so far. You stop when you find that the sum of the ratings of the last movie you have chosen and the movie with the highest rating among all the previous movies is greater than 7.5.

(a) Define an appropriate Markov chain and use the first step equations in order to find the expected number of movies you will have to choose.

(b) Now assume that the ratings of the movies are uniformly distributed in the interval $[0, 5]$. Write the first step equations for the expected number of movies you will have to choose in this case.

3. **Customers in a Store**

Consider two independent Poisson processes with rates λ_1 and λ_2. Those processes measure the number of customers arriving in store 1 and 2.

(a) What is the probability that a customer arrives in store 1 before any arrives in store 2?

(b) What is the probability that in the first hour exactly 6 customers arrive, in total, at the two stores?

(c) Given that exactly 6 have arrived, in total, at the two stores, what is the probability that exactly 4 went to store 1?

4. **Arrival Times of a Poisson Process**

Consider a Poisson process $(N_t, t \geq 0)$ with rate $\lambda = 1$. For $i \in \mathbb{Z}_{>0}$, let S_i be a random variable which is equal to the time of the i-th arrival.

(a) Find $\mathbb{E}[S_3 \mid N_1 = 2]$.

(b) Given $S_3 = s$, where $s > 0$, find the joint distribution of S_1 and S_2.

(c) Find $\mathbb{E}[S_2 \mid S_3 = s]$.

5. **Bus Arrivals at Cory Hall**

Starting at time 0, the 52 line makes stops at Cory Hall according to a Poisson process of rate λ. Students arrive at the stop according to an independent Poisson process of rate μ. Every time the bus arrives, all students waiting get on.

(a) Given that the interarrival time between bus $i-1$ and bus i is x, find the distribution for the number of students entering the ith bus. (Here, x is a given number, not a random quantity.)

(b) Given that a bus arrived at 9:30 AM, find the distribution for the number of students that will get on the next bus.

(c) Find the distribution of the number of students getting on the next bus to arrive after 9:30 AM, assuming that time 0 was infinitely far in the past.

6. **[Bonus] Choosing Two Good Movies (cont.)**

The bonus question is just for fun. You are not required to submit the bonus question, but do give it a try and write down your progress.

Solve the first step equations that you derived in Part (b), in order to find the expected number of movies that you will have to choose.