Electrical Engineering 126: Probability & Random Processes

Midterm 1 Cheat Sheet

Spring 2019

1 Distributions

- $X \sim \text{Bernoulli}(p)$, $p \in [0,1]$.

  PMF: $p_X(x) = p^x(1-p)^{1-x}$, $x \in \{0,1\}$.

  MGF: $M_X(s) = 1 - p + p\exp s$.

  Moments: $\mathbb{E}[X] = p$, $\operatorname{var} X = p(1-p)$.

- $X \sim \text{Binomial}(n,p)$, $n \in \mathbb{Z}_+$, $p \in [0,1]$.

  PMF: $p_X(x) = \binom{n}{x} p^x(1-p)^{n-x}$, $x \in \{0,\dots,n\}$.

  MGF: $M_X(s) = (1 - p + p\exp s)^n$.

  Moments: $\mathbb{E}[X] = np$, $\operatorname{var} X = np(1-p)$.

- $X \sim \text{Geometric}(p)$, $p \in (0,1)$.

  PMF: $p_X(x) = pq^{x-1}$, $x \in \mathbb{Z}_+$, $q = 1-p$.

  MGF: $M_X(s) = (p\exp s)/(1 - q\exp s)$, $s < \ln(1/q)$.

  Moments: $\mathbb{E}[X] = p^{-1}$, $\operatorname{var} X = q/p^2$.

- $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$.

  PMF: $p_X(x) = \lambda^x \exp(-\lambda)/x!$, $x \in \mathbb{N}$.

  MGF: $M_X(s) = \exp(\lambda(\exp s - 1))$.

  Moments: $\mathbb{E}[X] = \lambda$, $\operatorname{var} X = \lambda$.

  X, Y independent, $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\mu) \implies X + Y \sim \text{Poisson}(\lambda + \mu)$.

- $X \sim \text{Uniform}[a,b]$, $a < b$.

  PDF: $f_X(x) = (b-a)^{-1}$, $x \in [a,b]$.

  MGF: $M_X(s) = (\exp(sb) - \exp(sa))/(s(b-a))$.

  Moments: $\mathbb{E}[X] = (a+b)/2$, $\operatorname{var} X = (b-a)^2/12$.

- $X \sim \text{Exponential}(\lambda)$, $\lambda > 0$.

  PDF: $f_X(x) = \lambda\exp(-\lambda x)$, $x > 0$.

  CDF: $F_X(x) = (1 - \exp(-\lambda x))\mathbb{1}_{\{x \ge 0\}}$.

  MGF: $M_X(s) = \lambda/(\lambda - s)$, $s < \lambda$.

  Moments: $\mathbb{E}[X] = \lambda^{-1}$, $\operatorname{var} X = \lambda^{-2}$.

- $X \sim \mathcal{N}(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma^2 > 0$.

  PDF: $f_X(x) = (\sqrt{2\pi}\sigma)^{-1}\exp(-(x-\mu)^2/(2\sigma^2))$.

  CDF: $F_X(x) = \Phi(x)$.

  MGF: $M_X(s) = \exp(\mu s + \sigma^2 s^2/2)$.

  Moments: $\mathbb{E}[X] = \mu$, $\operatorname{var} X = \sigma^2$.

  X, Y independent, $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$, $Y \sim \mathcal{N}(\mu_2, \sigma_2^2) \implies X + Y \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

2 Definitions & Equations

Tail Sum: For $X \ge 0$, $\mathbb{E}[X] = \int_0^\infty \mathbb{P}(X \ge x)\,\mathrm{d}x$.

Variance: $\operatorname{var} X = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$. Sum: $\operatorname{var}\sum_{i=1}^n X_i = \sum_{i=1}^n \operatorname{var} X_i + \sum_{i \ne j}\operatorname{cov}(X_i, X_j)$.

Covariance: $\operatorname{cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$.

Correlation: $\rho(X,Y) = \operatorname{cov}(X,Y)/\sqrt{(\operatorname{var} X)(\operatorname{var} Y)}$.

Order Statistics:
$f_{X^{(i)}}(x) = n\binom{n-1}{i-1} f(x)F(x)^{i-1}(1 - F(x))^{n-i}$.
$F_{X^{(i)}}(x) = \sum_{k=i}^n \binom{n}{k} F(x)^k(1 - F(x))^{n-k}$.

MGF: $M_X(s) = \mathbb{E}[\exp(sX)]$.

Law of total variance: $\operatorname{var}(X) = \operatorname{var}(\mathbb{E}[X|Y]) + \mathbb{E}[\operatorname{var}(X|Y)]$

Markov: For $X \ge 0$, $x > 0$, $\mathbb{P}(X \ge x) \le \mathbb{E}[X]/x$.

Chebyshev: For $x > 0$, $\mathbb{P}(|X - \mathbb{E}[X]| \ge x) \le (\operatorname{var} X)/x^2$.

Chernoff: For all x, $\mathbb{P}(X \ge x) \le (M_X(s))/e^{sx}$ for all $s > 0$ where the MGF is defined.