UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 13
Spring 2019

1. **Orthogonal LLSE**

   (a) Consider zero-mean random variables X, Y, Z such that Y, Z are orthogonal. Show that $L[X \mid Y, Z] = L[X \mid Y] + L[X \mid Z]$.

   (b) Show that for any zero-mean random variables X, Y, Z it holds that:

   $$L[X \mid Y, Z] = L[X \mid Y] + L\big[X \mid Z - L[Z \mid Y]\big]$$

2. **Gaussian Estimation**

   Let $Y = X + Z$ and $U = X - Z$, where X and Z are i.i.d. $\mathcal{N}(0, 1)$.

   (a) Find the joint distribution of U and Y.

(b) Find the MMSE of X given the observation Y, call this $\hat{X}(Y)$.

(c) Let the estimation error $E = X - \hat{X}(Y)$. Find the conditional distribution of E given Y.

3. **Joint Gaussian Probability**

Let $X \sim \mathcal{N}(1,1)$ and $Y \sim \mathcal{N}(0,1)$ be jointly Gaussian with covariance ρ. What is $\mathbb{P}(X > Y)$?