UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 11
Spring 2020

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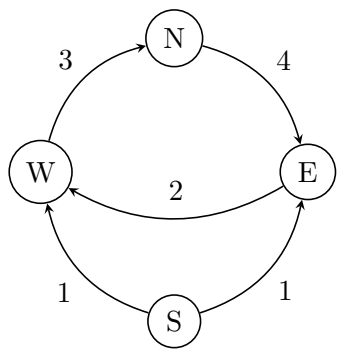
1. **Jump Chain Stationary Distribution**

   Use properties of transient states and the jump chain to find the stationary distribution of this CTMC.



2. **Sub-Critical Forest**

   Consider a random graph $\mathcal{G}(n, p(n))$ where $p(n) = o(\frac{1}{n})$ (this is called the **sub-critical phase**). Show that the probability that $\mathcal{G}(n, p(n))$ is a forest, i.e. contains no cycles, tends to 1 as $n \to \infty$.

   Hint: Let X_n be the number of cycles, and use Markov's inequality.

3. **Random Graph**

   Consider a random undirected graph on n vertices, where each of the $\binom{n}{2}$ possible edges is present with probality p independently of all the other edges. If $p = 0$ we have a fully empty graph with n completely disconnected vertices; in contrast, if $p = 1$, every edge exists, the network is an n-clique, and every vertex is a distance one from every other vertex.

   (a) Fix a particular vertex of the graph, and let D be a random variable which is equal to the degree of this vertex. What is the PMF of D? Calculate $\lambda \triangleq \mathbb{E}[D]$.

   (b) Assume that $c = np$ is a constant, independent of n. For large values of n, how you would approximate the PMF of D?