UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Session IX
Spring 2020

Problem 1. **Expected Squared Arrival Times**
Let $(N(t), t \geq 0)$ be a Poisson process with arrival instants $(T_n, n \in \mathbb{N})$, where $0 < T_1 < T_2 < \cdots$. Find $\mathbb{E}(\sum_{k=1}^{3} T_k^2 \mid N(1) = 3)$.

Problem 2. **Minimum and Maximum of Exponentials**
Let $\lambda_1, \lambda_2 > 0$, and $X_1 \sim \mathit{Exponential}(\lambda_1)$, $X_2 \sim \mathit{Exponential}(\lambda_2)$ are independent. Also, define $U := \min(X_1, X_2)$ and $V := \max(X_1, X_2)$. Show that U and $V - U$ are independent.

Problem 3. **Spatial Poisson Process**

A two-dimensional Poisson process of rate $\lambda > 0$ is a process of randomly occurring special points in the plane such that (i) for any region of area A the number of special points in that region has a Poisson distribution with mean λA, and (ii) the number of special points in non-overlapping regions is independent. For such a process consider an arbitrary location in the plane and let X denote its distance from its nearest special point (where distance between two points (x_1, y_1) and (x_2, y_2) is defined as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$). Show that:

(a) $\mathbb{P}(X > t) = \exp(-\lambda \pi t^2)$ for $t > 0$.

(b) $\mathbb{E}[X] = \dfrac{1}{2\sqrt{\lambda}}$.