Each time a fair coin is tossed it comes up H (heads) or T (tails) with equal probability, independently from time to time.

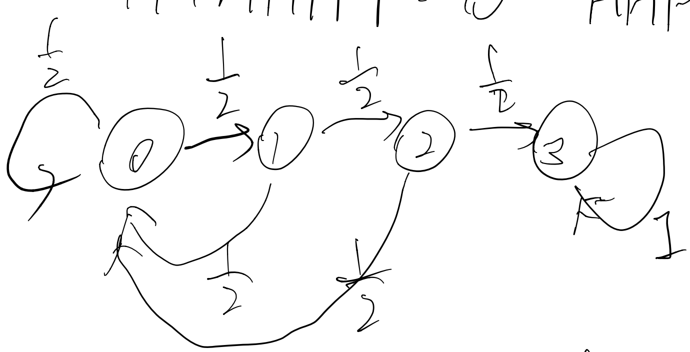
Find the expected number of times the coin needs to be tossed till we first see the pattern HHH (three successive heads). For example, in the pattern $HTHHTHTHTHHH$ we had to toss the coin 12 times till we first saw the pattern HHH.

Note: Even though the probability of seeing HHH in a sequence of three coin tosses is $\frac{1}{8}$, the mean number of tosses needed to see HHH is not 8.

Hint: The problem can be solved by considering a Markov chain with 4 states.

State 0, 1, 2, 3 : # largest contiguous string of heads

HTHHHT : 0   HHH : 3



mean time to reach 3

$h(i)$ = expected time to arrive at 3 starting from state i.

FSE:

$$
\begin{cases}
h(0) = 1 + \frac{1}{2} h(0) + \frac{1}{2} h(1) \\
h(1) = 1 + \frac{1}{2} h(2) + \frac{1}{2} h(0) \\
h(2) = 1 + \frac{1}{2} h(3) + \frac{1}{2} h(0) \\
h(3) = 0
\end{cases}
$$

$$\Rightarrow \quad h(0) = 14.$$

$$\frac{1}{8} \Rightarrow 8$$

HHH