1. **Machine**

A machine, once in production mode, operates continuously until an alarm signal is generated. The time up to the alarm signal is an exponential random variable with parameter 1. Subsequent to the alarm signal, the machine is tested for an exponentially distributed amount of time with parameter 5. With probability $1/2$ the test results are positive, in which case the machine returns to production mode. With probability $1/2$ the test results are negative, in which case the machine is taken for repair. The duration of the repair is exponentially distributed with parameter 3, after which the machine goes back to production.

(a) Let states $1$, $2$, $3$ correspond to production mode, testing, and repair, respectively. Let $(X_t)_{t \geq 0}$ denote the state of the system at time $t$. Is $(X_t)_{t \geq 0}$ a CTMC?

(b) Find the rate matrix $Q$ of the CTMC and transition matrix $P$ of the corresponding jump chain.

(c) Find the stationary distribution of the CTMC.

2. **Lazy Server**

Customers arrive at a queue in a service facility according to a Poisson process of rate $\lambda > 0$. The queue has infinite capacity. There is an infinitely powerful but lazy server who visits the service facility according to a Poisson process of rate $\mu > 0$. These two Poisson processes are independent. When the server visits the facility she instantaneously serves all the customers that are in the queue and then immediately leaves (until her next visit).

Thus, for instance, at any time, any customers that are waiting in the queue would only be those that arrived after the most recent visit of the server.

(a) Model the queue length as a CTMC and find the stationary distribution.

(b) Suppose that the CTMC is at stationary, find the mean number of customers waiting in the queue at any given time.

3. **Jump Chain Stationary Distribution**

Use properties of transient states and the jump chain to find the stationary distribution of this CTMC.

![Diagram of the CTMC with states W, S, E, N and transitions labeled with rates 1, 2, 3, and 4.]