UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 5
Spring 2021

1. Confidence Interval Comparisons

In order to estimate the probability of a head in a coin flip, p, you flip a coin n times, where n is a positive integer, and count the number of heads, S_n. You use the estimator $\hat{p} = S_n/n$.

(a) You choose the sample size n to have a guarantee

$$\Pr(|\hat{p} - p| \geq \epsilon) \leq \delta.$$

Using Chebyshev Inequality, determine n with the following parameters. Note that you should not have p in your final answer.

(i) Compare the value of n when $\epsilon = 0.05$, $\delta = 0.1$ to the value of n when $\epsilon = 0.1, \delta = 0.1$.

(ii) Compare the value of n when $\epsilon = 0.1, \delta = 0.05$ to the value of n when $\epsilon = 0.1, \delta = 0.1$.

(b) Now, we change the scenario slightly. You know that $p \in (0.4, 0.6)$ and would now like to determine the smallest n such that

$$\Pr\left(\frac{|\hat{p} - p|}{p} \leq 0.05\right) \geq 0.95.$$

Use the CLT to find the value of n that you should use. *Recall that the CLT states that the sum of IID random variables tends to a normal distribution with the sample mean and variance as it's parameters for n large enough.*

2. Convergence of Exponentials

Let $X_1, X_2, \ldots$ be i.i.d. Exponential(λ) random variables. Show that

$$\frac{X_n}{\ln n} \to 0 \qquad \text{in probability as } n \to \infty.$$

3. Breaking a Stick

I break a stick n times, where n is a positive integer, in the following manner: the ith time I break the stick, I keep a fraction X_i of the remaining stick where X_i is uniform on the interval $[0, 1]$ and $X_1, X_2, \ldots, X_n$ are i.i.d. Let $P_n = \prod_{i=1}^{n} X_i$ be the fraction of the original stick that I end up with.

(a) Show that $P_n^{1/n}$ converges almost surely to some constant function.

(b) Compute $\mathbb{E}[P_n]^{1/n}$.