UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 10
Spring 2022

1. Machine

A machine, once in production mode, operates continuously until an alarm signal is generated. The time up to the alarm signal is an exponential random variable with parameter 1. Subsequent to the alarm signal, the machine is tested for an exponentially distributed amount of time with parameter 5. The test results are positive, with probability 1/2, in which case the machine returns to production mode, or negative, with probability 1/2, in which case the machine is taken for repair. The duration of the repair is exponentially distributed with parameter 3.

(a) Let states $1, 2, 3$ correspond to production mode, testing, and repair, respectively. Let $(X(t))_{t \geq 0}$ denote the state of the system at time t. Is $(X(t))_{t \geq 0}$ a CTMC?

(b) Find the rate matrix Q of the CTMC and the transition matrix P of the corresponding jump chain.

(c) Find the stationary distribution of the CTMC.

2. Continuous-Time Markov Chains: Introduction

Consider the continuous-time Markov process with state space $\{1, 2, 3, 4\}$ and the rate matrix

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

(a) Find the stationary distribution p of the Markov process.

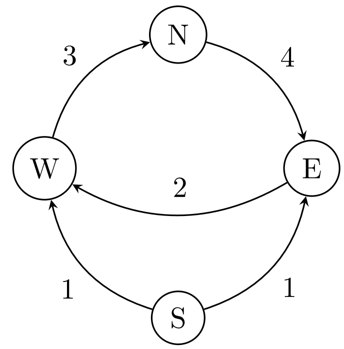
(b) Find the stationary distribtion π of the jump chain, i.e., the discrete-time Markov chain which only keeps track of the jumps of the CTMC. Formally, if the CTMC $(X(t))_{t \geq 0}$ jumps at times $T_1, T_2, T_3, \ldots$, then the DTMC is defined as $(Y_n)_{n=1}^{\infty}$ where $Y_n := X_{T_n}$.

(c) Suppose the chain starts in state 1. What is the expected amount of time until it changes state for the first time?

(d) Again assume the chain starts in state 1. What is the expected amount of time until the chain is in state 4?

3. Jump Chain Stationary Distribution

Use properties of transient states and the jump chain to find the stationary distribution of this CTMC.



4. Reversibility of CTMCs

We say a CTMC with rate matrix Q is *reversible* if there is a distribution p satisfying the detailed balance equations:

$$p_i q_{ij} = p_j q_{ji} \quad \forall i, j.$$

Show that if a CTMC is reversible w.r.t. p, then p is a stationary distribution for the chain. Furthermore, show that in this case the embedded chain is also reversible. *Remark.* The converse is true too, i.e. the CTMC is reversible given that the embedded chain is reversible.