UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 11
Spring 2022

1. **Generating Erdös-Renyi Random Graphs**

   True/False: Let G_1 and G_2 be independent Erdös-Renyi random graphs on n vertices with probabilities p_1 and p_2, respectively. Let $G = G_1 \cup G_2$, that is, G is generated by combining the edges from G_1 and G_2. Then, G is an Erdös-Renyi random graph on n vertices with probability $p_1 + p_2$.

2. **Random Graph**

   Consider a random undirected graph on n vertices, where each of the $\binom{n}{2}$ possible edges is present with probality p independently of all the other edges. If $p = 0$ we have a fully empty graph with n completely disconnected vertices; in contrast, if $p = 1$, every edge exists, the network is an n-clique, and every vertex is a distance one from every other vertex.

   (a) Fix a particular vertex of the graph, and let D be a random variable which is equal to the degree of this vertex. What is the PMF of D? Calculate $\lambda \triangleq \mathbb{E}[D]$.

   (b) Assume that $c = np$ is a constant, independent of n. For large values of n, how you would approximate the PMF of D?

3. **Exponential: MLE & MAP**

   The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X.

   (a) Find $\text{MLE}[X \mid Y]$.

   (b) Find $\text{MAP}[X \mid Y]$.

4. **Gaussians and the MSE**

   Suppose you draw n i.i.d. data points $(x_1, y_1), \ldots, (x_n, y_n)$, where n is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, Y has a linear dependence on X, with additive Gaussian noise.) Show that finding the MLE estimate of W given the data points $\{(x_i, y_i) : i = 1, \ldots, n\}$ is equivalent to minimizing the cost function

$$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2$$