1. Generating Erdős-Renyi Random Graphs

True/False: Let $G_1$ and $G_2$ be independent Erdős-Renyi random graphs on $n$ vertices with probabilities $p_1$ and $p_2$, respectively. Let $G = G_1 \cup G_2$, that is, $G$ is generated by combining the edges from $G_1$ and $G_2$. Then, $G$ is an Erdős-Renyi random graph on $n$ vertices with probability $p_1 + p_2$.

2. Random Graph

Consider a random undirected graph on $n$ vertices, where each of the $\binom{n}{2}$ possible edges is present with probability $p$ independently of all the other edges. If $p = 0$ we have a fully empty graph with $n$ completely disconnected vertices; in contrast, if $p = 1$, every edge exists, the network is an $n$-clique, and every vertex is a distance one from every other vertex.

(a) Fix a particular vertex of the graph, and let $D$ be a random variable which is equal to the degree of this vertex. What is the PMF of $D$? Calculate $\lambda \triangleq E[D]$.

(b) Assume that $c = np$ is a constant, independent of $n$. For large values of $n$, how you would approximate the PMF of $D$?

3. Exponential: MLE & MAP

The random variable $X$ is exponentially distributed with mean 1. Given $X$, the random variable $Y$ is exponentially distributed with rate $X$.

(a) Find $\text{MLE}[X \mid Y]$.

(b) Find $\text{MAP}[X \mid Y]$.

4. Gaussians and the MSE

Suppose you draw $n$ i.i.d. data points $(x_1, y_1), \ldots, (x_n, y_n)$, where $n$ is a positive integer and the true relationship is $Y = WX + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. (That is, $Y$ has a linear dependence on $X$, with additive Gaussian noise.) Show that finding the MLE estimate of $W$ given the data points $\{(x_i, y_i) : i = 1, \ldots, n\}$ is equivalent to minimizing the cost function

$$J(w) = \sum_{i=1}^{n} (y_i - wx_i)^2$$