UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

**Discussion 12**
Spring 2022

1. **Random Graph Estimation**

   Consider a random graph on $n$ vertices in which each edge appears independently with probability $p$. Let $D$ be the average degree of a vertex in the graph. Compute the maximum likelihood estimator of $p$ given $D$. You may approximate $\text{Binomial}(n, p) = \text{Poisson}(np)$.

2. **Flipping Coins and Hypothesizing**

   You flip a coin until you see heads. Let

$$X = \begin{cases} 1 & \text{if the bias of the coin is } q > p. \\ 0 & \text{if the bias of the coin is } p. \end{cases}$$

   Let $Y$ be the number of flips until we see heads. Find a decision rule $\hat{X}(Y)$ that maximizes $\mathbb{P}[\hat{X} = 1 \mid X = 1]$ subject to $\mathbb{P}[\hat{X} = 1 \mid X = 0] \leq \beta$ for $\beta \in [0, 1]$. Remember to calculate the randomization constant $\gamma$.

3. **Hypothesis Testing for Uniform Distribution**

   Assume that

   - If $X = 0$, then $Y \sim \text{Uniform}[-1, 1]$.
   - If $X = 1$, then $Y \sim \text{Uniform}[0, 2]$.

   Using the Neyman-Pearson formulation of hypothesis testing, find the optimal randomized *decision rule* $r : [-1, 2] \to \{0, 1\}$ with respect to the criterion

$$\begin{aligned} \min_{\text{randomized } r:[-1,2]\to\{0,1\}} \quad & \mathbb{P}\big(r(Y) = 0 \;\big|\; X = 1\big) \\ \text{s.t. } & \mathbb{P}\big(r(Y) = 1 \;\big|\; X = 0\big) \leq \beta, \end{aligned}$$

   where $\beta \in [0, 1]$ is a given upper bound on the false positive probability.

   (Note: We will be following the notation used in Walrand and lecture where the probability of false alarm (PFA) is bounded by $\beta$ as opposed to $\alpha$ used in the course notes.)