UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 01
Spring 2024

1. **Borel–Cantelli Lemma**

   If $A_1, A_2, \ldots$ is a sequence of events with $\sum_{i=1}^{\infty} \mathbb{P}(A_i) < \infty$, then

   $$\mathbb{P}(\text{infinitely many of } A_1, A_2, \ldots \text{ occur}) = 0.$$

   Remark: later we will see how Borel–Cantelli may be used to show some laws of large numbers.

2. **Independence**

   Events $A, B \in \mathcal{F}$ are said to be **independent** if $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$.

   a. Show that if events A, B are independent, then the probability exactly one of the events occurs is

   $$\mathbb{P}(A) + \mathbb{P}(B) - 2\,\mathbb{P}(A)\,\mathbb{P}(B).$$

   b. Show that if the event A is independent of itself, then $\mathbb{P}(A) = 0$ or 1.

3. **Puffcaps**

Consider a deck of 40 cards. Puffcaps are traps which are planted on cards in the deck and activated when drawn. At the beginning of the game, 40 puffcaps are planted on random cards, uniformly at random and independently of each other. A card may have multiple puffcaps planted on it. Given that Axel has drawn 20 cards and already activated 20 puffcaps, what is the probability that the next card he draws activates zero puffcaps?